# SIGMA

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# The New Maths

The importance of a knowledge of mathematics in any sphere of activity cannot be overemphasized. Social requirements are such that at least an elementary knowledge of arithmetic is essential. This importance is stressed by mathematics being a compulsory subject at the secondary level of education.

The New Maths curriculum was introduced in order to cater to the needs of a fast changing society and to keep pace with technological advances. While this is still in the experimental stage, we note that students, parents and teachers have been disappointed with this course. This could be due to:

(1) Inadequate training of staff resulting in a very superficial knowledge of the subject. Consequently the teachers are unable to present the subject with necessary clarity.

(2) The New Maths curriculum was presumably drawn up without due consideration for the local conditions.

The training programme and the development of the curriculum to suit our social needs should be carried out by "A training and research centre", set up for this purpose, consisting of personnel with a comprehensive knowledge of the subject. The most suitable personnel would be qualified mathematicians and experienced teachers.

# THE PLANETS ARE NOT ENOUGH

## ARTHUR C. CLARKE

Altogether apart from its scientific value, space travel has one justification that transcends all others. It is probably the only way we can hope to answer on of the supreme questions of philosophy: Is Man alone in the Universe? It seems incredible that ours should be the only inhabited planet among the millions of worlds that must exist among the stars, but we cannot solve this problem by speculating about it. If it can be sloved at all, it will be by visiting other planets to see for ourselves.

The Solar System, comprising the nine known worlds of our Sun and their numerous satellites, is a relatively compact structure,a snug little celestial oasis in an endless desert. It is true that millions of miles separate Earth from its neighbors, but such distances are cosmically trivial. They will even be trivial in terms of human engineering before another hundred years— a mere moment in historical time— have elapsed However, the distances that sunder us from the possible worlds of other stars are of a totally different order of magnitude, and there are fundamental reasons for thinking that nothing — no scientific discovery or technical achievement — will ever make *them* trivial.

When today's chemical fuels have been developed to the ultimate, and such tricks as refueling in space have been fully exploited, we will have spaceships which can attain speeds of about ten miles a second. That means that the moon will be reached in two or three days and the nearer planets in about half a year. (I am deliberately rounding these numbers off, and anyone who tries to check my arithmetic had better remember that spaceships will never travel in straight lines or a' uniform speeds.) The remoter planets, such as Jupiter and Saturn, could be reached only after many years of travel, and so the trio Moon-Mars-Venus marks the practical limit of exploration for chemically propelled spaceships. Even for these cases, it is all too easy to demonstrate that hundreds of tons of fuel would be needed for each ton of payload that would make the round trip.

This situation, which used to depress the pre-atomic-energy astronauts, will not last for long. Since we are not concerned here with engineering details, we can take it for granted that eventually nuclear power, in some form or other, will be harnessed for the purposes of space flight. With energies a millionfold greater then those available from chemical fuels, speeds of hundreds, and ultimately thousands, of miles a second will be attainable. Against such speeds, the Solar System will shrink until the inner planets are no more than a few hours apart, and even Pluto will be only a week or two from Earth. Moreover, there should be no resonable limit to the amount of equipment and material that could be taken on an interplanetary expedition. Anyone who doubts this may ponder the fact that the energy released by a single H-bomb is sufficient to carry about a million tons to Mars. It is true that we cannot as yet tap even a fraction of that energy for such a purpose, but there are already hints of how this may be done.

The short-lived Uranium Age will see the dawn of space flight; the succeeding era of fusion power will witness its fulfillment. But even when we can travel among the planets as freely as we now travel over this Earth, it seems that we will be no nearer to solving the problem of man's place in the Universe. That is a secret that will still lie hidden in the stars.

All the evidence indicates that we are alone in the Solar System. True, there is almost certainly some kind of life on Mars, and possibly on Venus—perhaps even on the Moon. (The slight evidence for lunar vegetation comes from the amateur observers who actually *look* at the Moon, and is regarded skeptically by professional astronomers, who could hardly care less about a small slag heap little more than a light-second away.) Vegetation, however, can provide little intellectual companionship Mars may be a paradise for the botanist, but it may have little to interest the zoologist– and nothing at all to lure the anthropologist and his colleagues across some scores of millions of miles of space.

This is likely to disappoint a great many people and to take much of the zest out of space travel. Yet it would be unreasonable to expect anything else; the planets have been in existence for several billion years, and only during the last 0001 per cent of that time has the human race been slightly civilized. Even if Mars and Venus have been (or will be) suitable for higher forms of life, the chances are wildly against our encountering beings anywhere near our cultural or intellectual level at this particular moment of time. If rational creatures exist on the planets, they will be millions of years ahead of us in development — or millions of years behind us. We may expect to meet apes or angels, but never men.

The angels can probably be ruled out at once. If they existed, then surely they would already have come here to have a look at us. Some people, of course, think that this is just what they are doing. I can only say that they are going about it in a very odd manner.

We had better assume, therefore, that neither on Mars nor Venus, nor on any other of the planets, will explorers from Earth encounter intelligent life. We are the only castaways upon the tiny raft of the Solar System, as it drifts forever along the Gulf Streams of the Galaxy.

This, then, is the challenge that sooner or later the human spirit must face, when the planets have been conquered and all their secrets brought home to Earth. The nearest of the stars is a million times farther away than the closest of the planets. The spaceships we may expect to see a generation from now would take about a hundred thousand years to reach Proxima Centauri, our nearest stellar neighbour. Even the hypothetical nuclear - powered spaceships which a full century of atomic engineering may produce could hardly make the journey in less than a thousand years.

The expressive term 'God's quarantine regulations' has been used to describe this state of affairs. At first sight, it appears that they are rigorously enforced There may be millions of inhabited worlds circling other suns, harboring beings who to us would seem godlike, with civilizations and cultures beyond our wildest dreams But we shall never meet them, and they for their part will never know of our existence.

So run the conclusions of most astronomers, even those who are quite convinced that mere common or garden interplanetary flight is just around the corner. But it is always dangerous to make negative predictions, and though the difficulties of *interstellar* travel are stupendous, they are not insuperable. It is by no means certain that man must remain trapped in the Solar System for eternity, never to know if he is a lonely freak of no cosmic significance,

There are two ways in which we might gain direct knowledge of other stellar systems without ever leaving our own. Rather surprisingly, it can be shown that radio communication would be perfectly feasible across interstella space, if very slow-speed telegraphy were employed. However, we can hardly assume that anyone would be listening in at the precise frequency with a receiver tuned to the extremely narrow band that would have to be employed. And even if they were, it would be extremely tedious learning to talk to them with no initial knowledge of their language—and having to wait many years for any acknowledgment of our own signals, as the radio waves came limping back across the light-years. If we sent a question to Proxima Centauri, it would be almost nine years before any answer could reach Earth.

A more practical, though at first sight more startling, solution would be to send a survey ship—unmanned. This would be a gigantic extrapolation of existing techniques, but it would not involve anything fundamentally new. Imagine an automatic vessel crammed with every type of recording instrument and controlled by an electronic brain with preset instructions. It would be launched out across space, aimed at a target it might not reach for a thousand years. But at last one of the stars ahead would begin to dominate the sky, and a century or so later, it would have grown into a sun, perhaps with planets circling around it. Sleeping instruments would wake, the tiny ship would check its speed, and its sense organs would start to record their impressions. It would circle world after world, following a program set up to cover all possible contingencies by men who had died a thousand years before. Then, with the priceless knowledge it had gained, it would begin the long voyage home.

This type of proxy exploration of the universe would be slow and uncertain and would demand long-range planing beyond the capacity of our age. Yet if there is no other way of contracting the stars, this is how it might be done. One millennium would make the investment in technical skill so that the next would reap the benefit. It would be as if Archimedes were to start a research project which could produce no results before the time of Einstein.

If men, and not merely their machines, are ever to reach the planets of other suns, problems of much greater difficulty will have to be solved. Stated in its simplest form, the question is this: How can men survive a journey which may last for several thousand years? It is rather surprising to find that there are at least five different answers which must be regarded as theoretical possibilities—however far they may be beyond the scope of today's science.

Medicine may provide two rather obvious solutions. There appears to be no fundamental reason why men should die when they do. It is certainly not a matter of the body 'wearing out' in the sense that an inanimate piece of machinery does, for in the course of a single year almost the antire fabric of the body is replaced by new material. When we have discovered the details of this process, it may be possible to extend the life span indefinitely, if so desired. Whether a crew of immortals, however well balanced and psychologically adjusted, could tolerate each other's company for several centuries in rather cramped quarters is an interesting subject for speculation.

Perhaps a better answer is that suggested by the story of Rip Van Winkle. Suspended animation (or, more accurately, a drastic slowing down of the body's metabolism) for periods of a few hours is now, of course, a medical commonplace. It requires no great stretch of the imagination to suppose that, with the aid of low temperatures and drugs, men may be able to hibernate for virtually unlimited periods We can picture an automatic ship with its oblivious crew making the long journey across the interstellar night until, when a new sun was looming up, the signal was sent out to trigger the mechanisms which would revive the sleepers. When their survey was completed, they would head back to Earth and slumber again until the time came to awake once more, and to greet a world which would regard them as survivors from the distant past.

The third solution was, to the best of my knowledge, suggested over thirty years ago by Professer J. D· Bernal in a long out-of-print essay, *The World, the Flesh, and the Devil*, which must rank as one of the most outstanding feats of scientific imagination in literature. Even today, many of the ideas propounded in this little book have never been fully developed, either in or out of science fiction. (Any requests from fellow authors to borrow my copy will be flatly ignored.)

Bernal imagined entire societies launched across space, in gigantic arks which would be closed, ecologically balanced systems They would, in fact, be miniature planets, upon which generations of men would live and die so that one day their remote descendants would return to Earth with the record of their celestial Odyssey.

The engineering, biological, and sociological problems involved in such an enterprise would be of fascinating complexity. The artificial planets (at least several miles in diameter) would have to be completely self-contained and self-supporting, and no material of any kind could be wasted. Commenting on the implications of such closed systems, *Time* magazine's able, erudite science editor Jonathan Leonard once hinted that cannibalism would be compulsory among interstellar travellers. This would be a matter of definition; we crew members of the two - billion - man spaceship Earth do not consider ourselves canaibals despite the fact that every one of us must have absorbed atoms which once formed part of Caesar and Socrates, Shakespeare and Solomon.

One cannot help feeling that the interstellar ark on its thousand-year voyages would be a cumbersome way of solving the problem, even if all the social and psychological difficulties could be ouercome. (Would the fiftieth generation still share the aspirations of their Pilgrim Fathers who set out from Earth so long ago?) There are, however, more sophisticated ways of getting men to the stars than the crude, brute-force methods outlined above. After the stars hardheaded engineering of the last few paragraphs, what follows may appear to verge upon fantasy. It involves, in the most fundamental sense of the word, the storage of human beings. And by that I do not mean anything as naive as suspended animation.

A few months ago, in an Australian laboratory, I was watching what appeared to be perfectly normal spermatozoa wriggling across the microscope field. They *were* perfectly normal, but their history was not. For three years, they had beed utterly immobile in a deep freeze, and there seemed little doubt that they could be kept fertile for centuries by the same technique What was still more surprising, there had been enough successes with the far larger and more delicate ova to indicate that they too might survive the same treatment. If this proves to be the case, reproduction will eventually became independent of time.

The social implications of this make anything in ***Brave New World*** seem like child's play, but I am not concerned here with the interesting results which might have been obtained by, for example, unititing the genes of Cleopatra and Newton, had this rechique been available earlier in history. (When such experiments are started, however, it would be as well to rememb3r Shaw's famous rejection of a similar prososal: 'But suppose, my dear, it turns out to have my beauty and your brains?'')[1]

The cumbersome interstellar ark, with its generations of travelers doomed to spend their entire lives in empty space, was merely a device to carry germ cells, knowledge, and culture from one sun to another. How much more efficient to send only the cells, to fertilize them automatically some twenty years before the voyage was due to end, to carry the embryos throuh to birth by techiques alrerdy foreshadowed in today's biology labs, and to bring up the babies under tutelage of cybernetic nurses who would teach them their inheritance and eheir destiny when they were capable of understanding it.

These children, knowing no parents, or indeed anyone of a different age from themselves, would grow up in the strange artificial world of their speeding ship, reaching maturity in time to explore the planets ahead of them—perhaps to be the ambassaders of humanity among alien races or perhaps to find, too late, that there was no home for them there. If their mission succeeded, it wourld be their duty (or that of their descendants, if the first generation could not complete the task) to see that the knowledge they had gained was someday carried back to Earth.

1 **We have Shaw's word for it that the would-be geneticist was a complete stranger and not, as frequently stated, Isadora Duncan.**

Would any society be morally justified, we may well ask, in planning so onerous and uncertain a future for its unborn —indeed unconceived – children? That is a question which different ages may answer in different ways. What to one era would seem a could-blooded sacrifice might to another appear a great and glorious advanture. There are complex problems here which cannot be settled by instinctive, emotional answers.

So far, we have assumed that all interstellar voyages must of necessity last for many hundereds or even thousands of years. The nearest star is more than four light-years; away; the Galaxy itself-the island Universe of which our Sun is one insignifican-member—is hundreds of thousands of light-years across; and the distances *between* the galaxies are of the order of a million light-years. The speed of light appears to be a fundamental limit to velocity; in this sense it is quite different from the now outmoded 'sound barrier,' which is merely an attribute of the particular gases which happen to constitute our atmosphere.

Even if we could reach the speed of light, therefore, interstellar journeys would still require many years of travel, and only in the case of the very nearest stars would it appear possible for a voyager to make round trip in a single lifetime, without resort to such techniques as suspended animation. However, as we shall see, the actual situation is a good deal more complex than this.

First of all, is it even theoretically possible to build spaceships capable of approaching the speed of light? (That is, 186,000 miles a second or 670,000,000 miles per hour.) The problem is that of finding a sufficient source of energy and applying it. Einstein's famous equation $E = mc^2$ gives an answer—on paper—which a few centuries of technology may be able to realize in terms of engineering. If we can achieve the total annihilation of matter—not the conversion of a mere fraction of a per cent of it into enegry – we can approach as near to the speed of light as we please. We can never reach it, but a journey at 99.9 per cent of the speed of light would, after all, take very little longer than one at exactly the speed of light, so the defference would hardly seem of practical importance.

Complete annihilation of matter as still as much a dream as atomic energy itself was thirty years ago. However, the discovery of the anti-proton (which engages in mutual suicide on meeting a normal proton) may be the first step on the road to its realization.

Traveling at speeds approaching that of light, however, involves us at once in one of the most baffling paradoxes which spring from the theory of relativity—the so-called 'time-dilation effect.' It is impossible to explain *why* this effect occurs without delving into very elementary yet extremely subtle mathematics. (There is nothing difficult about basic relativity math: most of it is simple algebra. The difficulty lies in the underlying concepts.) Nevertheless, even if the explanation must be skipped, the results of the time-dilation effect can be stated readily enough in nontechnical language.

Time itself is a variable quantity; the rate at which it flows depends upon the speed of the observer. The difference is infinitesimal at the velocities of everyday life, and even at the velocities of normal astronomical bodies. It is all important as we approach to within a few per cent of the speed of light. To put it crudely, the faster one travels, the more slowly time will pass. At the speed of light, time would cease to exist; the moment 'Now' would last forever.

Let us take an extreme example to show what this implies. If a spaceship left Earth for Proxima Centauri at the speed of light, and came back at once at the same velocity, it would have been gone for some eight and one-half years according to all the clocks and calenders of Earth. ***But the people in the ship, and all their clocks, would have recorded no lapsed time at all.***

At a physically attainable speed, say 95 per cent of the velocity of light, the inhabitants of the ship would think that the round trip had lasted about three years. At 99 per cent, it would have seemed little more than a year to them. In each case, however, they would return more than eight years — Earth time – after they had departed. (No allowance has been made here for stopping and starting, which would require additional time.)

If we imagine a more extensive trip, we get still more surprising results. The travelers might be gone for a thousand years, from the point of view of Earth, having set out for a star five hundred light-years away. If their ship had averaged 99.9 per cent of the speed of light, they would be fifty years older when they returned to an Earth – *where ten centuries had passed away!*[2]

It should be emphasized that this effect, incredible though it appears to be, is one of natural consequences of Einstein's theory. The equation connecting mass and energy once appeared to be equally fantastic and remote from any practical application. It would be very unwise, therefore, to assume that the equation linking time and

2 The physical riality of the time dilation effect has been the subject of unusually acrimonious debate in recent years. Very few scientists now have any doubt of its existence, but its magnitude may not have the values quoted above. My figures are based on special relativity, which is too unsophisticated to deal with the complexities of an actual flight.

velocity will never be of more than theoretical interest. Anything which does not violate natural laws must be considered a possibility—and the events of the last few decades have shown cleary enough that things which are possible will always be achieved if the incentive is sufficiently great.

Whether the incentive will be sufficient here is a question which only the future can answer. The men of five hundred or a thousand years from now will have motivations very different from ours, but if they are men at all they will still burn with that restless curiosity which has driven us over this world and which is about to take us into space. Sooner or later we will come to the edge of the Solar System and will be looking out across the ultimate abyss. Then we must choose whether we reach the stars —or whether we wait until the stars reach us.

# The Satellite - "Aryabhata"

**T. Thirukeswaran,** (B. Sc.,)

India joined the exclusive club of space explorers when an India-built satellite "ARYABHATA" was lofted to a height of about 600 km (372.8 miles) by a Soviet Intercosmos rocket from a Cosmodrome in the Soviet Union on Saturday the 19th of April this year. With this successful launching India became the ninth country to have her own satellite orbiting the earth.

It took 26 months to assemble the satellite and it cost approximately Rs. 50 million under the 1972 agreement, between the Indian Space Research Organisation (ISRC) and the USSR Academy of Science. The Indian Space Research Organisation (ISRC) formed by the late Dr. Sarabhaia initiated steps to obtain the know-how for satellite technology within the country by establishing the Satellite System Division (SSD), in 1970 as part of the Vikram Sarabhai space center (VSSC) in Trivandrum with Prof. U. R. Rao as its head. The offer of assistance extended by the Soviet Union in this direction was welcome and a series of discussions were held between the USSR Academy of Science and the ISRO. According to the agreement signed the ISRO was to design and build the satellite to be launched from the USSR using an inter Cosmos Vehicle. The actual implemetation of this task was taken up by Prof. S. Dhawan who took charge of the ISRO in July 1972. The Indian Scientific Satellite Project (ISSP) was set up -under the direction of Prof. U. R. Rao at Peenya near Bangalore.

The two primary objectives of the satellite preject are,

(1) To build indigenous capability in satellite technology and

(2) to conduct worth-while scientific experiments in space. The satellite which is an oblate spheroid, weighs about 360 kgms. and measures 1.6 metres from end to end.

The satellite launched into a near-circular orbit of 600 km altitude and 5.1 degrees inclination orbits the earth once in every 96 minutes. The orbital life time of the space craft is estimated to be nearly 2½ years, although its operational life from the scientific stand point will only be about six months. Due to the limitation imposed by the quantity of gas carried in the spacecraft's spin-up system.

The satellite carries scientific equipment to study X-Rays, stars, neutrons, gamma rays from the sun, and electron and ultra violet rays in the earth's atmosphere. In addition, the satellite is equipped for the conduction of three scientific experiments for investigation in the fields of X-Ray Astronomy, Solar Physics and Aeronomy.

The ground telementry station and the extensive communication links between Bangalore, Sriharikota (near Madras) and Moscow have been set up by ISRO scientists at the SAAR center. The primary ground station for receiving data and commanding the satellite is located at the Sriharikota (SHAR) near Madras. The functioning of this entire ground station has also been tested using a helicopter-borne satellite model and timulating the maximum range that the satellite will have during its orbit, to ensure that the ground station can receive the telementred data from the satellite.

A second ground station has been built in association with the USSR Academy of Science in Moscow for receiving the data from the Satellite. To further increase the data coverage, the French National Space Agency with whom the ISRO has co-operative arrangements, has been requested to provide the real time telementry reception and tracking of the satellite from a station of the French Space network.

In a develeping country like India which is essentially agriculure based' with its vast area and large population, purposeful and imaginative adoption of advanced technology like space Technology could initiate a total process of development in order to leap forward from a stage of backwardness and poverty.

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# දියුණුවෙන විද්‍යාවෙන් මිනිසා කොයි බටද ?

සෑම තැනකම කොයි කවුරුත් වගේ අද කථා කරන දෙයක් තමයි, විද්‍යාවේ දියුණුවේ ඵලවලින් මිනිසා සොයන විමුක්තිය ලද හැකිද කියා. මේ පිළිබඳව විශාල සැකයක් පහලවී තිබේ. දැනට අවුරුදු 25 කට පෙරට වඩා අද දශ ගුණයකින් විද්‍යාව දියුණු වී තිබෙනව එයින් ඔබව වැටහෙනව ඇති කොයිතරම් සීඝ්‍රලෙස දියුණු වෙනවද කියා නමුත් මිනිස් සිතේ ඇති විය යුතු සතුට මේ තරම් ඉක්මනින් වැඩි වීමක් පෙන්නුම් කරන්නෙ නැත. එම නිසා මිනිසා සොයන පතන ඒ සතුටම ලඟා වීමට අමාරු වී ඇත

මෙය තවත් නිපුණ ඇසකින් බැලීමට සංඛ්‍යානයෙන් දියුණු රටක් ගැන සලකමු. අපේ මිනුම් දණ්ඩ වශයෙන් ස්කැන්ඩිනේවියාව ගත හොත්, එහි අධික පොහොසතුන් සංඛ්‍යාවක් දක්නට ලැබේ. දුප්පතුන් ඉතා විරලය. එහෙත් මෙතෙක් සැප පහසුකම් මැද වුවත් සිය දිවි නසා ගැනීම් ආදිය වැඩි වීමෙන් අසහනය වැඩි විම එහි පෙන්නුම් කෙරේ. එයින් අපගේ සාධාරණ සැකය තවත් වැඩි වේ මිනිසාට වඩා දියුණු වෙන් අඩු තිරිසන් සතුන් පතන සතුට කුමක් උවත් ඔවුන් ප්‍රීතියෙන් ජීවත් වන බව කිව හැක.

සිත තුල ආදරය, ක්‍රෝධය, කරුණාව, මෛත්‍රිය වෛරය යනාදිය අඩංගු වේ මෙම සිත ශරීරයෙන් වෙන් කල නොහැක. සෑම කරදරයකදීම ශරීරයත් සිතත් එකට මුහුණ දේ. අධ්‍යාත්මයේ භාහිර කොටසට ශරීරය පිනවීම ගැනේ එහිදී පෙර මතු ජීවිත ගැන නොසලකමු. එසේ වන විට ලැබිය යුතු සතුට මිනිසා මෙලොව දීම ලද යුතුය සතුට සැප පහසුකම් මත රැඳේ නම් එයට විද්‍යාව බොහෝ ලෙස සලකා ඇත. ගමනාගමනය දියුණුය හොඳ ඇඳුම් පැලඳුම් වර්ග නිපැදී ඇත. ආහාර වර්ග පහසුවෙන් ගැනීමට හැක. ඇස, කන, දිව සිරුර පිනා වීමට බොහෝ දේ නිපැද වී ඇත ඒත් අසහනය වැඩි වී ඇත මෙහි වරද විද්‍යාවේ දියුණු වේද? නැත සැප පහසු කම් කොයි තරම් තිබුනත් සිත කණගාටු වෙන් නම් එම සප නොදනේ. එසේම සතුටු සිතට කරදර නොතේරේ. සතුට අසතුට මත රැඳෙන සංතය විද්‍යාවේ දියුණුවෙන් පමණක්ම ලද නොහැක දුන් සිදුවන්නේ සැප සමග සතුට ඇති නොවීමයි. ක්‍රමක්‍රමයෙන් සතුට ලබා ගැනීමට සිතේ තිබෙන ශක්තිය අඩු වී යයි. සිත යම් සැපතක් සඳහා ඊයේ සතුටු වූවානම් අද ඒ සතුට ලැබීමට ඊට වඩා වැඩි සැපතක් ගැන වෙහෙසෙයි හෙට ඒ සතුට ලැබෙන්නේ ඊටත් වඩා සැපතක් ලැබීමෙනි. මේ තරම් සැප පහසු කම් විද්‍යා දියුණුවෙන් ලැබෙද්දීත් නියම සතුට ලබාගත නොහැක්කේ සතුට ලබාගැනීමට සිතේ ඇති හැකියාව දින දින අඩු වීම නිසාය.

මෙම සතුට ලබා ගැනීමේ හැකියාව නොවෙනස්ව තබා ගැනීමට සැප පහසුකම් සීමා කල යුතුය. එහෙත් එයින් දුක් විදීමක් අදහස් නොකෙරේ ස්වභාවික ආහාර වලට ප්‍රිය වීමෙන් සත්ව මාංශ වලින් දිවේ රස උරා ගැනීමේ හැකියාව අඩු වීම වලක්වා ගත හැක. මිනිසාගේ අධ්‍යාත්මය පිරිහෙමින් පවතී. මිනිසාට ප්‍රිය උපද වන දේ විද්‍යාවෙන් නිපැදේ. නමුත් ඊට වඩා හොඳ දෙයක් දුටු විට පරණ නිපැයුමෙන් ලැබෙන සතුට අඩුවේ අලුත් දේ ලබාගැනීමට සිතෙන විට පරණදේ ලබාගැනුමෙන් ඇතිවෙන සතුට අඩුවේ. ලද දෙයින් සතුටු වෙමු යන්න මෙහිදී අදහස් නොකෙරේ. මිනිසා දියුනුවන විට අලුත් දෙයට ආශාව ඇති වේ. විද්‍යාවේ දියුනුව නොනවතින දෙයකි එම නිසා නොනවතින අසහනයක් ඇතිවේ

මෙයින් මිදීමට අලුත් භාණ්ඩවල නිෂ්පාදනය සීමා කිරීම කල හැක වරක් මෙය රුසියාවේ සිදුකෙරින භාහිර රටවලට තිබෙන සම්බන්ධකම් නිසා එය අසාර්තක විය. නමුත් දැන් රුසියාවේ අලුත්දේ නිපැදවෙයි. චීනයෙත් මෙසේම විය තවත් අතකින් දැන්වීම් නතර කලහොත් මෙහි පාලනයක් දකිය හැක නමුත් සිත ඇතුලේ ආසාව නිතරම අලුත් දෙයකට යොමුවේ. අනෙක් අය භාවිතා කරන දේවල් වලට කැමති නැති අතර තමාට විශේෂ කැපී පෙනෙන දේ අවශ්‍ය වේ. කුඩා ලමයකු පවා වඩා උස පනින්න කැමතිය.

නමුත් මේ දේවල් බලහත් කාරයෙන් නැවැත් විය නොහැක. එම නිසා මෙසේ වීමට හේතුව අවබෝධ කර ගැනීමෙන් පමණක් එක් තරා විධියක සැනසුමක් ලබාගත හැක. එවිට මිනිසාගේ සතුට ආරක්‍ෂාවේ.

මේ ගැන ඔබ ටිකක් සිතා බලන්න මිනිසා මේ ගැන නොසිතා අනුකරයෙන් වැරදි මාවතක ගමන් කරයි. එය මනුෂ්‍ය සංහතියෙ විනාශයටම හේතුවිය හැක. විනාශය යන අදහස භාර ගැනීමෙන් අශානයට තවත් සාදකයක් එකතුවේ. අමාරුවෙන් හෝ මේ ගැන හරි ලෙස අවබෝධයක් ලබා ගැනීමෙන් සතුට ඇතිවේ. මෙහි අදහස තාපස ජීවිතයක් නොවේ. ඉදුරන් නැවීමත් සතුටට හේතු වන්නේ නැත.

අනවශ්‍ය නිපදවීම් වැලැක්වීම තවත් හොඳ අදහසකි. පොඩි ක්‍රීඩාවක් ලෙස දවසක් විශාල සාප්පුවක් වෙත ගොස් එහි ඇති අත්‍යවශ්‍යදේ කොපමණද අනවශ්‍ය දේ කොපමණද තෝරන්න අනවශ්‍ය එක් දෙයක් වශයෙන් පිට කසා ගැනීමට ඇති විද්‍යුත් උපකරණයක් ගැදිත් විය හැක. ඕනැකම්ත්ම චාම් දිවියක් ගෙන යාමත් සිතට දුක ඇති කරවයි. නමුත් සීමිත ලොවක අසීමිත දේ නිපදවීමට නොහැක. උදාහරණ වශයෙන් යකඩ තෙල් ආදිය.

සිතේ ඇතිවන ආශාව, කාමය මර්දනය කිරීමෙන් සිතට දුක් කරදරත් ශක්තිය විනාශ වීමත් සිදුවේ. අවශ්‍ය වන්නේ මර්දනය නොව හරි දැකීමයි. මර්දනය කිරීමෙන් විකෘතිය උපදී.

කාමයට, ආශාවට යටත් වීමවත්, ඒවා මර්දනය කිරීමවත් අවශ්‍ය නොවන අතර හරි දැකීමක් තිබිය යුතු වේ. එසේ නොවුව හොත් විනාශය අත ලඟය. කෙසේ වෙතත් විද්‍යාවේ දියුණුව මිනිසා සතුටට පත්කරයි. මිනිසාට සැපත සතුට ගෙන දෙන ලෙස එය තවත් ඉදිරියට ඇදෙනු ඇත මේම දියුණුව නොදන්නා කමින් වෙනත් අතකට හරවා ගැනීමෙන් මිනිසා නරක මාවතක ගමන් කරයි. මේ මිනිස් විනාශය ගැන තැති ගැනීම මිනිසා තුල තවම ඇති වී නැත. එය දැනුන දවසට මිනිස් සිත තවත් ව්‍යාකූල වේවි. විද්‍යා දියුණුව ලොවට අවශ්‍ය වෙනවා වගේම එයින් නිසි ප්‍රයෝජන ලබා ගැනීමට නම් මිනිසා හරි දැක්මකින් හරි අවබෝධයකින් එය දිහා බැලිය යුතුව ඇත.

(ආචාර්ය ඊ. ඩබ්ලිව්. අදිකාරම් මහතාගේ කථාවක් ඇසුරෙන් සකස් කලේ .කේ. කේ. ධර්මතිලක)

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**BOOK REVIEW:** සාපේක්ෂතා වාදය යනු කුමක් ද?

# Einstein in Sinhala*

***by Dr Valentine Joseph***

Einstein's name is synonymous with Relativity, and relativists can hardly forget the warm and strong personality that was Einstein.

A relativist who attempts a popular exposition of relativity often founders on the laughter of the gods. The irony of it all is that the founder of the theory tried his hand in the art of popularization, but failed. There has been a flood of popular books on relativity in the last three decades. Professor L. Landau, Nobel Laureate for Physics, of Moscow University and his colleague Professor Yu Rumer took up the challenge and brought out a book entitled "What is Relativity" in 1959. The book is remarkable for its originality and lively cartoons. The subject is treated with sympathy and understanding, It reflects in many ways the social significance of science. and points to the fact that knowledge is a form of perception of collective man. The book was translated into English by N. Kemmer F.R.S. in 1960. Einstein has now turned up dressed in Sinhala. He looks quite smart in the translator's outfit.

The words space and time have different shades of meaning in different cultural groups. This is quite inevitable, since space and time are forms of perception, and are invariably steeped in the cultural aspirations of the people. In a preface to the English translation of his book a distinguished continental relativist writes:

"The French speak of l' espace and of le temps. The use of the article le implies the conception of time as some mythological Chrones eating up his own children or a roller crushing the living present to dead past. It is an advantage that space and time are used without an article. The English philosophical minds are less liable to take them for existing entities and more ready to see that events do not occur in pre-established time and space . I praise the language which admits of such a formulation". Wonder what the Frenh, German and

Russians have to say about! It is claimed that there are as many conceptions of space and time as there are cultures. Nevertheless, relativists all over the world are agreed on the mathematical structure of space-time whatever approach each one of them may adopt in presenting the subject. Einstein seems to have bequeathed the human race with the spirit of unity amidst diversity.

The teaching of relativity is beset with grave dangers. The heart of the problem lies in the **allusion** to reality made by the mathematical language on the one hand and by the living (human) language on the other. One deals with the substratum of the world while the other describe phenomenon. The crucial word simultaneous has been translated into sinhal a as **එකවිට, එකවර එකවේලාව, එකමොහොත, සමකාලය,** and **සමකාලික** Other important terms in relativity such as relative and absolute have been translated into **සාපේක්ෂ** and **නිරපේක්ෂ** while space and the time take the familiar forms **අවකාශය** and **වේලාව** It is a pity that the translator has not even bothered to write a preface for the book. Explanatory notes on space and time as they appear in our culture, would have gone a long way to enliven the book for the sinhala reader. In a subject such as relativity much would depend on the ability of the reader to find out for himself the semantic meaning of each sentence. There are some minor errors in the text, where reference is made to either the wrong cartoon or the wrong page. Besides these short comings, the book is readable.

The book is not meant for the gullible who devour cheap science fiction. It is addressed to the thoughtful student who is familiar with the elementary process of reasoning. Needless to say, popular books can only give the reader an inkling of what the subject is all about. By no stretch of the imagination can it ever bring the reader face to face with the true Jacob. But such books have immense pedagogical value. The sinhala student from the backwoods of this isle has been denied good scientific literature. He has had to remain content with a few text-books which contain useful information but provide little or no insight into the nature of things. The book is bound to rouse the curiosity of the student. Since the cultural implications of relativity have not been fully fathomed, the student may in the course of time rediscover his own cultural identity in the subject. If the students asks for more after reading it, the book will have fulfilled its purpose. The book is a 'must' for every school library.

In the history of human thought, relativity is as important as the discovery that the earth is round. In mathematical language, this discovery amounts to the 'symbolic' unearthing of a three dimensional sphere of negative curvature called velocity space. Professors Landau and Rumer have tried to bring home the spirit of the subject in a cartoon which appears at the very end of their book. It is a delightful caricature of the naive and uninitiated.

***Part of this article appeared in the CDN** of August 30th 1974.

Fig. I
In the original text

Fig. II
In the sinhala version

## The Relativity of Direction !

Those who find it difficult to accept relativity or fear to venture into deeper waters are very much like the helpless people in the cartoon (Fig. I)

But the printer's devil (Fig II) in the sinhala version has created a gem of an idea. The result is hilarious! Far from disfiguring the text, it provokes thought and enriches the very theme of the cartoon... relativity of direction. Professor Landau would have been delighted to see the very last cartoon in the sinhala version of his now famous book.

As man the homo sapien stumbles in the dark spaces of his own being, in search of a momentary footing on the external world, he can hear the laughter of the gods.

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**"Let us remember, please, that the search for the constitution of the world is one of the greatest and noblest problems presented by nature."**

— G. GALILEI

# STATISTICAL MEASUREMENT OF WASTAGE IN EDUCATION

— DR. J. KERR —

This paper is an attempt to show how some of the data collected in the Annual School Census may be used in a quantitative analysis of wastage at the Primary and Junior Secondary levels of education.

Wastage within an Educational System occurs in many forms. The most conveniently measured forms are dropping out and repetition, although it is recognised that other aspects (e. g. inefficient use of teachers, classrooms and laboratories; pupil and teacher absenteeism) are also prevalent and elimination of these may be necessary to reduce the incidence of dropping out and repetition. Repetition refers to pupils who spend a further year in the same grade doing the same work and dropping out is pupils leaving the system before the completion of a given school cycle. The limitations in the use of the concept of drop-outs and repeaters will be discussed later. Inefficiency resulting from the failure to recruit all children into the school system and from unemployed school leavers is beyond the scope of this paper.

The School Census gives the total enrolment and number of repeaters for each grade; the number of drop-outs can easily be deduced from these figures,

**Example:**

Grade I

| | *Enrolment* | *Repeaters* |
|---|---|---|
| Year 1 | 100 | 20 |
| Year 2 | 110 | 24 |

Grade II

| | *Enrolment* | *Repeaters* |
|---|---|---|
| Year 1 | 80 | 15 |
| Year 2 | 82 | 18 |

i.e. in Year 1, 20 of the 100 pupils enrolled in grade I were repeating the grade. 100 pupils were enrolled in Grade I in year 1, of these 24 pupils repeated the grade the following year and a further 64 = 82—18 pupils were promoted to grade II. Consequently the remaining 12 pupils must have dropped out of Grade I by the end of year 1. Similarly, given the Grade III enrolments, the drop-outs from grade II may be found etc. Calculation of drop-outs by this method have been found to be more reliable than asking each school for the number of drop-outs from each grade (It is difficult for schools to distinguish between pupils who drop-out and those that leave to transfer to another school). It is usual to express the numbers of promoters, drop-outs and repeaters as proportions of the original enrolment, these figures are called respectively the promotion (p), drop-out (d) and repetition (r) rates.

The drop-out and repetition rates themselves give only a limited picture of the wastage within an Educational System; the construction of a cohort flow diagram provides information on the cumulative-effects of drop-out and retention and also in the resources spent for each successful completer of a given school cycle.

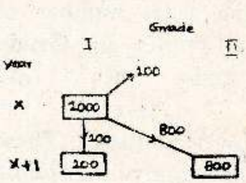
The standard method of constructing a cohort flow diagram is to apply the appropriate drop-out repetition and promotion rates to a theoretical cohort of 1000 (say) pupils enrolled in Grade I in Year X and hence determine the numbers who will be in Grades I and II in year X+1. The appropriate rates for the year X+1 are then used to continue the progress of the surviving pupils. This procedure is then repeated until all 1000 pupils have either completed the cycle or dropped out at some stage.

Again, a simple example will illustrate the method.

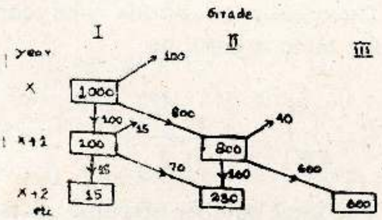
Consider the following rates for the years X and X+1.

| | Grade I | | | Grade II | | |
|---|---|---|---|---|---|---|
| | p | r | d | p | r | d |
| X | 80 | 10 | 10 | | | |
| X+1 | 70 | 15 | 15 | 75 | 20 | 5 |

At the start of year X+1, 800 pupils will hence be promoted to Grade 2, 100 will be still in Grade 1 and the remaining 100 will have left school. This may be portrayed in a flow diagram:-




Then, applying the respective rates for the year X+1 to the above, produces the following diagram:-




Although this reconstructed cohort method has been adopted by most developing countries it is not completely satisfactory, but it can be improved without the collection of any additional data.

The main drawback is that, to be able to construct a flow diaram for a 5 year Primary Cycle, drop-out, repetition and promotion rates must be known for at least eight successive years. Even if such figures are available, then the subsequent analysis of wastage is for a cohort enrolled in grade 1 eight years previously; whereas educational planners are much more concerned with the present situation and what is likely to happen in the future. One way to solve this problem is to investigate what would happen to a theoretical cohort, currently enrolled in Grade 1, if the present drop-out repetition and promotion rates persist in future years (i.e. the same rates p-are used for any particular grade for all years in the construction of the flow diagram). The drop-out, repetition and promotion rates may be estimated either from the latest known figures or, if in recent years the rates for each grade have been fairly construent, then a single estimate can be obtained by combining the figures for the past four or five years. Unfortunately at present insufficient data is available to allow the use of the latter method in Sri Lanka.

All the above methods assume that the repetition and drop-out rates for pupils newly promoted to a grade are identical to those for pupils repeating the grade. No attempt has so far been made to test the validity of this assumption.

The completed flow diagram, whatever its construction can then be used to obtain several standard indicators of wastage.

(1) The percentage of pupils who complete the cycle, or inversely the percentage who drop-out of the cycle.

(2) The Input - out-put Ratio. This is the ratio of the number of pupil years spent per successful completer to the prescribed duration of the cycle.

(3) The percentage of pupil years wasted by

(a) completers and (b) drop-outs.

(a) This is a number of pupil years the completers spent in excess of the prescribed duration of the cycle.

(b) This is the total numbers of pupil-years spent by drop-outs, it may be sub-divided into effective (leading to promotion) and non-effective (leading to repetition and drop-out) pupil years.

The indicators should be used for within a country rather than between a country comparisons; one country may have a perfectly efficient educational system that caters for only ten percent of the school age population whereas in another country all children may be enrolled in school at some stage but the system may be very inefficient.

There are two obvious faults attached to these standard indicators; drop-outs are considered a complete waste, however many grades have previously been completed, and a repeated year is assumed to be a wasted year. Certainly drop-outs who have successfully completed only one or two years will have gained little from their stay in school, but this is certainly not the case for drop-outs from Grades 4 or 5. The problem is how much value do such drop-outs obtain from their education compared to successful completers of the primary cycle. Repetition is not regarded as an opportunity for pupils to recover an earlier failure. rather it is seem as wasteful as it leads to overcrowding of classrooms and higher educational costs.

**Evaluation of wastage in schools in,**
**a) Colombo Region,**

**(b) the whole island, using the results of the school census for 1972 & 73.**

If the improved reconstructed cohort method is to be used, it is not necessary to construct an actual flow diagram to evaluate the wastage indicators; simple formulae can be found for all the relevant quantities.

Let pi, ri and di be respectively the promotion, repetition and drop-out rates for grade i. Now consider in new enrolments to Grade i (not necessarily all in the same year). The total number of three pupils eventually promoted can be shown to be $\frac{np_i}{(1-r_i)}$ and consequently a total of $\frac{nd_i}{(1-r_i)}$ pupils would drop-out of Grade i.

Further the total number of pupil years spent by all pupils in Grade i would be $\frac{n}{(1-r_i)}$ and $\frac{np_i}{(1-r_i)^2}$ of those would be spent by promoters. These results can be combined to give the following Formulae for a theoretical cohort of 1000 pupils currenty enrolled in Grade 1 of a 5 year Primary Cycle.

(1) The number of pupils who complete the cycle is given by

$$1000 \times \prod_{r=1}^{5} \frac{p_i}{(1-r_i)}$$

(2) The total number of pupil-years spent by the cohort equals

$$1000 \sum_{j=1}^{5} \frac{p_1 \, p_2 \ldots p_{j-1}}{(1-r_1)\,(1-r_2) \ldots (i-r_j)} \qquad p_o = 1$$

The number of pupils dropping-out from Grade i is

$$1000 \, \frac{p_1 \, p_2 \ldots p_{i-1} \, d_i}{(1-r_1)\,(1-r_2) \ldots \ldots (1-r_i)}$$

(3) The total number of pupil–years spent by successful complets of the cycle is

$$1000\left(\prod_{i=1}^{5}\frac{p_i}{(1-r_i)}\right)\left(\sum_{i=1}^{5}\frac{1}{(1-r_i)}\right)$$

**Table 1**

## Promotion, Repetition and Drop-out Rates 1972-3

| Grade | COLOMBO | | | ISLAND | | |
|---|---|---|---|---|---|---|
| | p | r | d* | p | r | d |
| 1 | ·751 | ·229 | ·020 | ·666 | ·294 | ·040 |
| 2 | ·791 | ·172 | ·037 | ·726 | ·221 | ·053 |
| 3 | ·791 | ·172 | ·037 | ·741 | ·206 | ·053 |
| 4 | ·791 | ·155 | ·054 | ·756 | ·180 | ·064 |
| 5 | ·817 | ·119 | ·064 | ·774 | ·153 | ·073 |

* Guessed Estimates.

Substituting these figures into the above formulae produces the following results:–

**(a) Colombo**

Evolution of the cohort

| 1 | | 2 | | 3 | | 4 | | 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1000 | → | 974 | → | 931 | → | 889 | → | 832 | → | 772 |
| ↓ 26 | | ↓ 43 | | ↓ 42 | | ↓ 57 | | ↓ 60 | | |

The total number of pupil-years spent by the cohort = 5594 of which
4654 were spent by completers
940 by drop–outs.

The number of effective (i.e. leading to promotion) pupil years for completers was 3860 and 538 for drop–outs.

Conseqently the number of wasted years (leading to repetition or drop-out) was 794 for completers and 402 for drop–outs.

The total number of pupil–years spent in excess equals 5594 — 3860 = 1734 of which 54·2% were attributable to drop–outs and 45·8% to completers; however, 31·0% of excess years were effective for drop-outs.

7·25 pupil years were required for each completer of the cycle compared with the prescribed duration of 5 years, giving an Input/Output ratio of 1.45.

**(b) Whole Island**

Evolution of the cohort

| 1 | | 2 | | 3 | | 4 | | 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1000 | → | 943 | → | 879 | → | 820 | → | 756 | → | 691 |
| ↓ 57 | | ↓ 64 | | ↓ 59 | | ↓ 64 | | ↓ 65 | | |

the 691 pupils successfully complete the Primary Cycle.

The total number of pupil-years spent by the cohort = 5628 of which 4396 were spent by completers; 1232 by drop-outs.

The number of effective pupil-years was 3455 for completers, 634 for drop-outs. Consequently the number of wasted years was 941 for completers and 598 for drop-outs. The total number of pupil years spent in excess equals 5628 — 3455 = 2173 of which 56·7% were attributable to drop-outs, 43·3% to completers; however, 29·2% of excess years were effective for drop-outs. 8·14 pupil-years were required for each completer of the cycle, giving an Input/Output ratio of 1.63.

**Comments**

As we would expect the extent of wastage is less in Colombo than the rest of the Island, one interesting result is that, although the Colombo cohort had more successful completers of the cycle than the Island cohort, the total number of pupil-years spent was less in Colombo. However these comparisions depend on the accuracy of the figures in Table 1. The Island rates should be reliable but the Colombo figures assume that there are no net transfers of pupils in or out of the Region during the period 1972/3. Unfortunately, no data on transfers was collected at that time.

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# The New Responsibilities of the Mathematics Department

***Professor V. K. Samaranayake***

The primary responsibility of the Mathematics Department has been and will be, as in all other departments in the Faculty of Natural Sciences, to conduct the special and general degree courses in Pure and Applied Mathematics, maintaining standards accepted internationally. The syllabi have been updated from time to time. This process together with the continued practice of appointing external examiners from leading British Universities, has allowed us to maintain standards accepted abroad. Although we will continue with these practices in the future, the department will also consider the compatibility of various subjects of study, the volume of work involved and the relevance for Sri Lanka. The stress on the applied aspects of Statistics and the introduction of practical classes has resulted from such considerations. The Department welcomes suggestions for changes to the curriculum and views on some of the proposals listed below, from students in particular.

A proposal that originated from the department to introduce Statistics as a subject for the General Degree in Natural Sciences has been approved by the Faculty. A limited number of students would be selected for this course from those who successfully complete the First Examination in Natural Sciences offering Pure Mathematics and Applied Mathematics, commencing 1976. This proposal needs the approval of the Senate and Vice Chancellor before it becomes effective. Once this proposal is approved, we also hope to suggest for consideration by Faculty and Senate, that those reading for the Special Degree in Mathematics be allowed to sit the General Degree as well offering Pure Mathematics, Applied Mathematics and Statistics as subjects, with he proviso that only one of the two degrees would be confered to successful candidates. This arrangement would be most helpful to a student who fails the special degree as he can fall back on the pass (or even class) eligible on his performance at the general degree examination. Other suggestions that are under discussion include the introduction of optional papers for the special degree and the inclusion of a project dissertation as part of the examination. Both suggestions are heavily dependent on an adequate number of senior staff for successful operation.

Another traditional responsibility of a University department is the development of postgraduate courses and research activity. The shortage of qualified staff, books and journals and the heavy teaching load has resulted in research being limited to individual projects by senior staff. They have been encouraged by short term visits to research institutes abroad. The International Centre for Theoretical Physics at Trieste, Italy set up in the 60's by the International Atomic Energy Agency (and now supported by UNESCO as well) to stop the exodus of qualified scientists from developing countries to institutions having better facilities for research, by granting short term fellowships tenable at the Centre. Regular visits to the centre where all requirements for research activity in Theoretical Physics is available, has helped some of us keep up with the recent trends sn a fast developing field.

In order to encourage our own Assistant Lecturers and others who have graduated recently to engage in research activities locally, we have introduced postgraduate Diploma courses in Mathematical Physics, Applied Statistics and Mathematics, to be followed by research leading to a Masters degree. We hope to form a nucleus of research students from amoung those who perform well at the Diploma Examination.

Our efforts to develop research activity requires external assistance. We have already solicited the support of the Mathematics staff of the other Campuses for the Diploma courses. Success will also depend on the availability of books, journals and contact with other research institutions. We believe that this can be achieved by the estabilishment of links between the department and institutions abroad. These links should provide for an exchange of staff, information on research trends, books and journals. Staff from the foreign institution would act as external examiners and external supervisors for research degrees. They would advice us on curriculum changes at the under graduate level as well. If the link is tied to an aid giving agency, then a modest grant could provide for the travelling expenses for the staff exchanges and for the purchase of books, journals and equipment.

Such a link has been proposed between the Department of Mathematics of the Colombo Campus and the Department of Applied Statistics of the University of Reading for the development of the Statistical Unit *into a full fledged Statistical Centre capable of successful involvement in the teaching, curriculum development, research and advisory work in Statistics. These proposals have been accepted in prin-

*The Statistical Unit of the Colombo Campus was established in 1969 as a result of simultaneous requests for such a unit that reached the Vice Chancellor of then University of Ceylon, Colombo from the Faculty of Medicine and Departments of Geography and Mathematics.

ciple by the University, the Government of Sri Lanka, the British Council and the Department of Applied Statistics of the University of Reading. They are now with the British authorities and we hope the link would be established before the end of the year. Our contact with the International Centre for Theoretical Physics can also be called a link for the development of Theoretieal Physics. With the commencement of the Diploma course in Mathematical Physics we hope to approach the IAEA for a more formal link with Trieste wlth possible aid for books and journals in addition to thefellowships already provided. We hope to form a similar line for Pure Mathematics with a University in France, Italy or the U.S.A.

Until recently probationary assistant lecturer were reluctant to register for research degrees locally for fear of not being able to undergo training abroad. Our attempts to encourage research activity locally will not jeopardise the chances of obtaining foreign scholarships and infact would help them score over other candidates. They would also have an opportunity of making use of the short visits envisaged under the link arrangements. A lecturer who has had some research experience locally would be in a better position to gain from working at a foreign institution than an inexperienced one who would have to spend almost one year in preparatory work. Some of us had to spend quite some time attending courses on background topics and in learning necessary techniques such as computer programming which can now be achieved here. We have conducted computer programming classes since 1968 with the last few being sponsored by the Mathematical and Astronomical Society. Since last year computer programming is part of the Mathematical Methods course for Part I. We have also established a computing service for other departments and clients of the Statistical Unit. We are fortunate to have access to the computers at the Department of Census and Statistics, the State Engineering Corporation and the Katubedda Campus. Our aim is to encourage students to obtain a working knowledge of a programming language before he leaves us. The proposed link with Reading will enable us to expand our programme library and also obtain a programmable calculator that could be used to teach programming techniques.

It was stated earlier that the link with Reading was for the development of the Statistical Unit of the Colombo Campus. It is necessary to explain the relation between the Unit and the Mathematics department. The Unit functions under the guidance of an Advisory Committee directed by its Chairman. All members of the department of Mathematics who are interested in Statistics are members of the Unit and its administration is handled by the department. During the past one year the Unit has progressed rapidly to become a very useful and active part of the Colombo Campus. The Unit is at present responsible for the teaching and curriculum development of the courses in Basic Mathematics for the Development Studies and Biologlaci Sciences degree courses and in Statistis for the degree courses in Special Mathematics, Applied Mathematics, Education and Development Studies. The Unit also conducts short courses on specialised topics during vacations the most recent being on Medical Statistics. In addition to the above the Unit provides a consultancy and data processing service in Statistics to University departments and other organisations. The consultancy work has brought us real life problems that are in some way or other vital to the socio economic development of Sri Lanka While actively contributing to the development effort by our involvement in a scientific study of these problems we also gain by the exposure of our staff and students to these problems. The Unit has prepared a series of Case studies on the use of Statistics in Sri Lanka. These are being used as course material for the Development Studies degree course.

The teaching and curriculum development work mentioned above are officially the responsibility of the department of Mathematies. The Unit acts as an agent of the department. The department has recruited a new category of teachers called Instructors in Mathematics to undertake most of the teaehing in the Arts and Education faculties. The Statistical Services job range of the Development Studies degree course is another responsiblity of the department. Through this job range we hope to produce graduates who can take up a position as a Statistical Officer or a Statistical Investigator without further training. We are also involved in producing Mathematics teachers through the Faculty of Education.

The departments main interest have been in Pure Mathematics, Mathematical physics and Statistics. Other areas that will become increasingly important are Numerical Methods, Computing and Mathematies Education. In recent years, as will be seen in this report, the fields that have developed most are Mathematical Physics and Statistics. This reflets the aid the depatment has received and the availability of staff, in these areas. We hope that during the next few years, ehanges of similar magnitude will take plaee in the other areas as well,

The enlarged activity of the Mathematics Department extending to three faculties neccesitated in creating a second chair which was filled recently. Campus and University authorities have been generous in granting an increased cadre and a reasonable equipment grant. We hope that it would be possible to successfuly fulfil our responsibilities, both old and new.

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# அணுக்கரு மின்கலங்கள்
# (Nuclear Batteries)

எழுதியவர்: ஆ. மகேந்திரசிங்கம்.
(பௌதிக சிறப்புநெறி இறுதியாண்டு)

இன்று பல ஆயிரம் உவோற் அளவுடைய மின்கலங்கள் ஆக்கப்பட்டுள்ளன. சாதாரணமாக நாங்கள் பயன்படுத்தும் டார்ச் விளக்கு மின்கலங்கள் (Torch Battery) அளவை கொண்டு, பல ஆயிரம் வோல்ட் மின்அழுத்தம் உடைய அணுக்கரு மின்கலங்கள் ஆக்கப்பட்டுள்ளன. ஆனல் இவை குறைந்த மின்னேட்டமும் (Low Current) உயர் மின்அழுத்தமும் உடைய (High Potential) மின்கலங்கள் ஆகும்.

அணுக்கரு மின்கலங்களின் தத்துவம் எளிது. இவற்றில் உபயோகிக்கப்படும் கதிர் இயக்க சமதானிகள் (Radio active Isotopes) பொலோனியம் -210 (அரை ஆயுட் காலம் -138 நாட்கள்) துரந்தியம் -90 (25-ஆண்டுகள்) ஆகியவை β-கதிர்வீசும் கதிர் இயக்க சமதானிகளே சிறந்தவை. இவற்றில் இருந்து வெளியாகும் சத்தி மின்சத்தியாக மின்கலங்களில் மாற்றப்படுகிறது. இவ்வாறு மாற்ற ''வெப்ப மின்விளைவு'' (Thermo electric effect) பயன்படுகிறது.

கதிர் இயக்க சமதானிகளில் இருந்து எழும் கதிர்கள் கவசமொன்றில் தாக்கினல் கதிர்வீசல் சத்தி (Radiation energy) வெப்ப

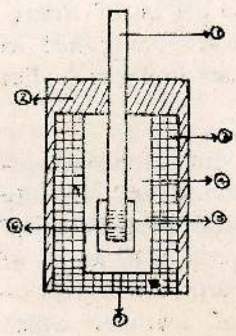
சத்தியாக மாறுகிறது. இந்த வெப்ப சத்தியை ஒரு வெப்பஇணை மின்சத்தியாக மாற்றுகிறது.

இம் மின்கலங்களின் அழுத்தம் அதிகமாக இருப்பினும் வலு (Power) குறைவானதே. இவை பாரமற்றவையாகவும் சிறியனவாகவும் இருப்பதால் எடுத்துச்செல்ல இடப்பெயற்சிக்கு இலகுவாகவும் இருக்கின்றன. பல்லாண்டு காலத்திற்கு இவை தொடர்ந்து மின்சத்தியை வழங்கவல்லன. அணுக்கரு மின்கலங்களின் கதிர்வீச்சை வெப்பம், அமுக்கம், அதிர்ச்சி போன்றவை பாதிக்க மாட்டாது.

அணுக்கரு மின்கலங்கள் இன்று பல துறைகளில் பயன்படுத்தப்படுகிறது. இராணுவத்திற்கும், விண்வெளி ஆராய்ச்சிக்கும் (Space research) இன்று முக்கியமாகப் பயன்படுகிறது. இடப்பெயற்சிக்கு எளிதாக இருப்பதால் இராணுவத்தினர் வானொலித்தொலைத் தொடர்பு (Radio Telecommunication) சாதனங்களுக்கு பயன்படுத்துகின்றனர். விண்வெளியில் செல்லும் ரொக்கெட், விண்வெளி கப்பல் (Space Ship) ஆகியவற்றில் கருவிகளை இயக்க அணுகரு மின்கலங்கள் பாவிக்கப்படுகின்றன. மேலும் தானியக்க காலநிலை அறிவிப்பு நிலையங்களிலும் (Weather Stations) கடல் அடியில் வைக்கப்படும் சாதனங்களிலும் இம் மின்கலங்களை பயன்படுத்துகின்றனர். ஆர்க்டிக் பசுபிக் கடலின் நடுவே கால நிலைகளை அறிவிக்க அணுக்கரு மின்கலங்களில் இயக்கும் கருவிகள் பயன்படுகின்றன. விண்வெளி கப்பலில் இருந்து கால நிலை, விண்வெளி கதிர்வீசல், காற்றின் அமுக்கம், வெப்பம் போன்றவை ரேடியோ அலைகளாக மாற்றப்பட்டு (Radiowaves) அது பரப்பப்படுகின்றன (weve transmission).

## அணுக்கரு மின்சாரத்தின் செயற்பாட்டுமுறை:

கதிர்வீசும் கதிர் இயக்க சமதானிகளில் இருந்து எழும் β–கதிர்களை மின்கோல் ஒன்றில் (electrode) சேகரித்து இன்னொரு மின்கோலை அதனின்றும் பிரித்து மின்கலத்தில் அமைத்து இருக்கிறார்கள். இரு மின்கோல்களுக்கும் (electrodes) இடையே காவலி (Insulator) பாலிஸ்டிரீன் (Polystyrene) பொருத்தப்பட்டிருக்கிறது. படத்தில் β–கதிர் வீசும் கதிர்இயக்க சமதானி ஸ்ரோன்சியம்-90 (Sr), ஏற்றியம்-90 ஆகியவற்றைக் கொண்டுள்ள அணுக்கரு மின்கலம் ஆக்கப்பட்டுள்ளது. இதன் குறி எண் B-50 இது ஐக்கிய அமெரிக்க நாடுகளில் தயாரிக்கப்படுகிறது. கதிர் வீசல் சமதானிகளாகிய ஸ்டரான்சியம்-90 ஏற்றியம்-90 இவை தங்க குழாய் ஒன்றில் வைக்கப்பட்டுள்ளன. Sr-90 இன் அரை ஆயுள் 25 ஆண்டுகள் சுரப்பியில் (source) இருந்து வரும் β-கதிர்களை அலுமினியக் குழாய் ஒன்று சேகரிக்கிறது. கதிர்வீசல் வெளியேறுதபடி ஈயக் கவசம் உள்ளது. மின் கோல்களாக தங்கம் (Gold) அனோட்டாகவும் (Anode) தாமிரம் கதோட்டாகவும் (Cathode) பயன்படுகின்றன. இருமின் வாய்களிற்கும் இடையே காவலி (Insulator) பாலிஸ்டிரின் (Polystyrene) வைக்கப்பட்டுள்ளது எதிர்மின்னுடைய β–துகள்கள் அலுமினிய சேகரியில் சேருகின்றன. மின்கோல்களை ஒரு கம்பியால் தொடுத்தால் மின்னோட்டம் உண்டாகிறது. மின்கோல்களிற்கிடையே உயர் மின்னழுத்தம் 6000V மின்னோட்டப் பூச்சிய வோல்டில் 40$\mu\mu$A மிக குறைந்த மின்னோட்டம் உடையன. வெப்பமும் அமுக்கமும் இம்மின்கலங்களை என்றும் பாதிக்க மாட்டாது. இவை நீண்டகாலம் பயன் தருபவை. Sr-90 இன் அரை ஆயுள்காலம் சுமார் 25 ஆண்டுகள். எனவே அணுக்கரு மின்கலங்கள் நீண்ட ஆயுள் உடையன!



1. தங்கத்தகடு நேர்முனை (anode)
2. இறுகிய பொருள்
3. ஈயக் கவசம்
4. பாலிஸ்ரின் மின் காவலி
5. அலுமினிய சேகரி
6. { ஸ்ரோன்சியம் 90 / ஏற்றியம் 90 } கதிர்வீசல் சமதானிகள்
7. தாமிரம் எதிர்முனை (cathode)

B–50 அணுக்கரு மின்கலம் உச்சமின் அழுத்தம் 6000 V உடைய அணுக்கரு மின்கலம். இது ஐக்கிய அமெரிக்க நாடுகளில் செய்யப்படுகிறது.

# WHITHER NEW MATHS?

*The following papers on the Current Situation in New Maths were presented at the Seminar "Whither New Maths" organized by the Department of Mathematics in Collaboration with the Mathematical and Astronomical Society of the Colombo Campus.*

*The speakers, Prof. P. D. Gunatileka (Katubedda Campus) Prof. K. A. Epasinghe, Mr. D. G. W. Gunapala (Vidyodaya Campus) Dr. V. Joseph, Dr. R. D. Stern (Colombo Campus) and Mr. D. A. Perera (Director, Curriculum Development Centre) were introduced by Prof. V. K. Samaranayake.*

*The Seminar was followed by a panel discussion which was chaired by Prof. C.R. Kulatileka. (Vidyalankara Campus).*

# Nature of Mathematics

**— Prof. P. D. GUNATILEKA —**

First recorded in the sixteenth century the word mathematics is of Greek origin. It was the collective name for geometry, arithmetic and certain physical sciences involving geometrical reasoning as astronomy and optics.

However, the question "what is mathematics"? cannot be answered meaningfully by philosophical generalities, semantic definitions or journalists. Circumlocutisms, just as such characterisation fails to do justice to music and painting. No one can from an appreciation of these arts without some experience with rhythm, harmony and structure of with form, colour and composition. For the appreciation of mathematics actual contact with its substance is even more necessary.

With this caution, some remarks of a general nature can nevertheless be made. Primarily mathematics is a method of inquiry known as postulational thinking. The method consists in carefully formulating definitions of the concepts to be discussed and in explicitly stating the assumptions that shall be the basis for reasoning. From these definitions and assumptions conclusions are deduced by the application of the most rigorous logic man is capable of using.

To describe mathematics as only a method of inquiry is to discribe Leonard da de Vimin's 'Last Supper' as an organisation of paint on canvas. Mathematics is also a field for creative endeavours. In deriving what can be proved, as well as in constructing methods of proof, mathematics employ a h gh order of intuition and imagination.

Over and above all other drives to create is the search for beauty.

To quote Bertrand Russell, the master of abstract mathematical thought.

"Mathematics rightly viewed possesses supreme beauty - a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the-georgeous trappings of painting or music, yet sublimly pure and capable of a stern perfection such as only the greatest art can show. The time spirit of delight, the exultation, the sense of being more than man which is the touchstone of the highest excellance is to be found in mathematics as surely as in poetry."

Kepler and Newton for example were men of wonderful imaginative power, which enabled them not only to break away from age-long and rigid tradition but also to set up new and revolutionary concepts.

If then, mathematics is indeed a creative activity what force causes men to pursue it? The most obvious though not necessarilly the most important motive for mathematical investigation has been to answer questions arising directly out of social needs. Commercial and financial transactions, navigation, calendar reckoning, the construction of bridges, dams, churches, tanks, temples and palaces, the design of fortifications and weapons of warfare and numerous other human pursuits involve problems that can best be resolved by mathematics. It is specially true of our engineering age that mathematics is a universal tool.

Another basic use of mathematics. indeed one that is especially prominent in modern times has been to provide a national organisation of natural phenomena. The concepts methods and concelusions of mathematics are the substraction of the physical sciences.

A method of inquiry, a creative endeavour, a substraction of physical sciences. an art with a beauty of its own if that is mathematics, what then is New Mathematics or briefly New Maths?

To answer this question we refer to the post-war history of Europe and the American Continent.

It was the best of times. Allied forces having won many battles in Europe, Asia and Africa were returning home to their fatherlands. In the United States of America, the returning military leaders, reputed among other things that they found U.S. fighting men deficient in mathematical skills.

This and the launching of a satellite by the Russians in 1957 gave a big push to new theories in science and mathematics education. In 1958 the National Science Foundation financed the largest project in the history of mathematical education, S.M.S.G. or the School Mathematics Stndy Group. A number of people—professors, teachers, psychologists and social scientists were brought together to revise, rewrite and revamp school mathematics.

Looking towards Europe of the same period one sees that new demands of mathematical skill were created as a result of the invention in Britain, of the electric digital computer – with its need for Boolean Algebra - a branch of mathematics which had lain neglected for a century. This discovery too gave impetus to the mathematical development in schools and Colleges of the United States of America, when there had already existed unrest about the low standard of mathematical attainment.

It was not long before the educational circles in Britain began to follow suit on their thinking. On the personal invitation of Dr. John M. Hammersely, a fellow of Trinity College, Oxford, a conference was commenced for the purpose of bringing together initially for the first time, those who taught mathematics in the schools and those who used mathematios in real life. This was the source of the School Mathematical Project – now commonly known as S.M.P.

As a result of work carried out by School Mathematics Study Group, School Mathematics Project and other similar organisations in both the United States of America and the United Kingdom, a new curriculum in mathematics has come into existence. Since some parts of this curriculum are younger in time than the tradi–

tional curriculum, it has assumed the somewhat misplaced title of "New Mathematics" which is often abbreviated to "New Maths" in U.S.A. and "New Maths" in U. K.

The "New Maths" is not new in the sense that it presents new concepts in the field of mathematics. The content of "New Maths" can be found in text books dating as far back as a hundred years or more.

To assess the significance of "New Maths" in the context of education in Sri Lanka it is important to compare the motivation for the introduction of New Maths in the West and their relevance to local conditions.

In the foreseeable future, I for one cannot envisage the possibility of our soldiers fighting extensive military battles either on native soil or in foreign lands.

We would at least be only spectators of any competitive activity in outer space. A massive and total take over of our society by technology of the twentieth century also reasons remote. Thus there seems hardly a panel between the motivations. Art led to school curriculum development in the West end that in Sri Lanka. Yet we are aware that New Maths is here with us today.

Looking at our own society and our culture, I cannot help but feel that there is a significant message in mathematics for us and for the future generations. And hence, school mathematics curriculum development is a matter of interest to all of us. It is a matter too important to be left in the hands of only a few educationalists.

In its broadest aspect mathematics is a spirit, the spirit of rationality. It is this spirit that challenges, stimulates, invigorates and drives human minds to exercise themselves to the fullest. It is this spirit that seeks to influence decisively the physical, moral and social life of men, that seeks to answer the problems possed by our very existence that strives to understand and construct nature and that exerts itself to explain and establish the deepest and utmost implications of knowledge already obtained.

May our deliberations here today be a step in the furtherence of this spirit.

# New Mathematics and the Local School System

Mr. D. A. PERERA

The pharse "new mathematics" may be used with different meanings. It may be used to refer to what is new in mathematics in the sense of mathematics invented by researchrs. It may refer to content which is new for a particular programme of teaching mathematics. "New" in this latter sense may refer to content which is quite old. In the former sense, there obiviously no new mathematics in the local school system. In the latter sense there is this article discribes very briefly the reasons for changing the mathematics content of the local school programmes.

One major reason is that the new content provides a better basis for the development of fundamental mathematical concepts, for example the concept of a variable. It is accepted that pupils should have a sound understanding of fundamental mathematical concepts. Under the earlier content what was emphasized was the mastery of techniques and the development of different types of skill to solve different types of problems. The problems were neatly classified. "X" was to be used only in the algebra class and not in the arithmetic or geometry class. The new content enables a teacher to relate these different areas and thus further pupil understanding and appreciation and promote better use of mathematics by them. Under the earlier content, given a simple equation to variables in the "equation solving class" pupils would have said that the equation cannot be solved because "to solve equations with two unknowns you should have two equations". Given the identical equation in the "graph drawing class" they would have quite happily drawn the tables for x and y and plotted a few points. with the new content there is emphasis an understanding what an equation is and pupils would learn that. whether an equation can be solved or not depends on the domain of the variables. The new content has not thrown out the old nor is new content like a new front to an old house. Pursuing This analogy further the old house has been pulled down, and the old material along with some new ones has been used to build a new house which is more functional. The new programme would enable more people to be more literate mathematically.

Another reason for introducing new content is that it enables very interesting fare to be provided to pupils, not only is the material interesting but the computational difficulty is much less and hence pupils tend to enjoy it more. Many adults recall their school mathematics with considerable dislike. The earlier mathematics was, by and large, ment for the very able few. In the primary classes children enjoy establishing, the relation "was born in", between the set of children in the class and the set of months of the year. This involves no computation at all. But it leads to very meanigful computation, largely of their own choice Because having drawn the mapping diagram they observe that more children are born in one month than another. Some may stop here, but others may count and say how many more. In the higher classes pupils enjoy drawing an incidence matrix for a given network. This again involves no computational difficulty. During this for different networks. They begin to see certain patterns in the incidence matrices which correspond to certain properties of the networks. They are making a mathematical discovery of their own. This delights them and further more gives them a sense of achievement that they are also "good" at mathematics. Under the eariler content not only did many fail ao enjoy mathematics but many were not good" at mathematics.

A third reason for introducing new content is that we in this country cannot keep aloof from the main stream of changes that are undoubtedly taking place in the world of school mathematics. During the past two decades many International conferences of mathematicians and mathematics educators have urged the inclusion of this new content into school mathematics. Recent Unesco Reports on the teaching of mathematics have advocated this. It is felt in some quarters that these changes are taking place only in the developed west. They would do well to study the mathematics tents for the secondary schools published by the National Council of Educational Research and Training of India, or the Report of the Conference on "Mathematics in Commonwealth Schools" held in 1968. It is true that "new mathematics" arose in the technologically advanced West. The reason given above should suffice to show that we need not wait (if at all that day is ever going to dawn) till we have all of that advanced technologically before, our children can give up using such books as a treatise on geometry written for adults by a mathematician, more than two thousand year ago.

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"We must learn to live together as brothers of perish together as fools." — **Martin Luther King, Jr.**

# The Psychological Basis for Curriculum Reform

D. G. W. GUNAPALA

In the developed countries the late nineteen fifties and the sixties saw the emergence of school curriculum reform as the main exercise of the time in the field of education. The demands of the knowledge expression, the scientific and tecnological revolution, the emergence of new values etc. and the educational problems inherent in these changes can be cited as the causes that led to these activities. This brought together various subject specialists, educational psychologists and practicing teachers to work as teams to review and revise the existing school curriculum with the aim of reforming it or mapping out something quite new.

In this short paper I wish to deal with some basic themes which have since then served as foundations of the various curriculum development programs and also influenced the thinking of academicians and educational specialists involved

1. Identifying the constituent disciplines and their key concepts.
2. The idea of the structure of a discipline.
3. New theories on the cognitive development of the child.
4. New findings in learning theory their implications.

1. Identifying the constituent disciplines and their key concepts.

The traditional School Curriculum had been a naturally evolving continuation of the Seven Arts which formed the content of teaching since the middle ages. Various elements had been added and others dropped out as the need arose but, basically the traditional Seven Arts formed the core. Reviews showed that in almost all subject areas there was much that was outmoded or irrelevant–That the content of some subject areas had been unchanged for the last hundred years. Much that was new which had been added to the pool of human knowled-during this country was not being taught at school. This was true for the area of mathematics also.

The need for a complete review and rethinking on new lines was suggested by Philip Phenix in 1958 in a book entitled Philosophy of Education. Making a complete philosophical analysis of the fields of human knowledge, he indicated the need to sift these in order to identify the relevant diciplines or subject areas and to highlight the essential features of each dicipline. This would help the teacher to place emphasis on central and definite matters in each discipline rather than teach a host of subordinate details the relevance of which may not be clear to the student. It was also shown that the use of key ideas in each discipline in the organisation and teaching of subject matter would lead to economy in learning effort, greatly increase the depth of comprehension and facilitate further exploration in any given discipline.

In the field of mathematics these ideas indicated the need for a number of changes, some of which are;

(1) Viewing the subject as an integrated Study as opposed to the compartmentalised form in traditional maths.

(2) Introduction to the subject through more fundamental concepts such as sets, relations etc. and laying emphasis on these basic concepts.

(3) Deleting of much tedious work involving mechanical calculations which often made the child lose sight of the basic mathematical ideas involved.

2nd theme

**The structure of a discipline**

The participation of university scholars in school curriculum development in those countries has had a significant effect in the development of the subject areas consistuting the school curriculum. It is such a subject specialist in a given area who would be most competent to identify the key concepts and also outline the interrelations between these key concepts and ideas, thus classifying the structure of a discipline. This emphasis on the structure of dicipline is another idea which has had a striking impact on the field of education. This new idea was put out forward by Jerome S. Bruner in 1960 in his book "The Process of Education". Writing about the structure as the fundamental element in learning subject matter he says,

Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully. To learn structure in short, is to learn how things are related". Illustrating this further, he indicated how understanding something as a specific instance of general case, i. e. as a general principle which could be applied in a number of situations helps the child to build up a model for understanding other similar situations that he may later encounter,

Mathematics, by its very nature is a highly structured subject. The study of mathematics essentially requires grasping of the basic concepts and the gradual building up of the higher order concepts and principles. Having decided upon the key concepts and ideas, as mentioned in the earlier section, all new mathematics programs attempt to build up meaningful relationships between these to arrive at higher order concepts and principles. Also, as far as possible the work is arranged in such a way that the child himself discovers and arrives at these relations and thus grasps the basic structure of the subject.

3rd theme

**New knowledge about the cognitive development of the child.**

In this respect findings of the Swiss Psychologist Jean Piaget has had the greatest impact in Shaping the new curricula both as to the sequencing of subject content as well as the approaches to the teaching.

For the last thirty to forty years Piaget has been gradually building up a theory on the growth of cognitive structures of the human being from infancy right upto the adult stage. This theory based upon a large number of carefully planned experiments on the development of mathematical and scientific concepts, growth of logical thinking and reasoning etc. has provided educationists with a clear picture regarding the cognitive activity of the Child which provides a psychological basis for proper planning of teaching and learning. Though these findings had been published some years back it was with the curriculum reform movement that his work was rediscovered and put to proper use in the educational field.

Piaget divides the process of Cognitive growth into four stages or period based upon the districtive features of each stage. These are, I Sensory Motor Period from birth to 2 yrs.

II Period of Pre-operations from 2 to 7 yrs.

III Period of concrete-operations from 7 yrs to 11 yrs.

IV Period of Formal-operations from 11 yrs to 15 yrs.

These age divisions are to be regarded as rough limits at which transitors occur from one period to the next. The important feature is that growth at each stage proceeds on the achievements of the preceding stages.

I will very briefly dwell upon some features of these stages which would be of relevance to our tasks here.

**I The Sensory Motor Period**

This deals with the features of the child activity in infancy where he manages to cope up with the challenges of the enviroment depending mainly on immediate sensory data and his own motor actions. This period is not of much importance to us here except to keep in mind that much of the dealings on a representative plane in later stages depend upon the sensory motor experiences of this period.

**II Period of pre-operations.**

This deals with the nature of thought of the pre-school child. Although he has started to use a language his basic concepts are only partially developed and incomplete. For example, Piaget has shown that although the child at this stage can count, use numbers, and can even be taught to mechanically add and subtract simple numbers, he has yet not fully developed the concept of uumber in the sense that an adult does. This comes only at the next stage i.e. concrete operational stage when he develops the ability of true classification and seriation, which form the basis for recognising the cardinal and ordinal properties of number.

The nature of the childs' thought at this stage can best be illustrated by an experiment Piaget has conductd on the conservation of quantity - The famous conservation experiment. In this experiment equal quantities of water were poured initially into two identical glass beakers A and B. The child was questioned as to the equality of the quantities of water and he readily agreed that they were so. Then in the presence of the child water in B was poured into a wider beaker C. and the child questionedagain as to the quantities of water in A and C. Though he was convinced that no water was added or Spilled out, the child replied that there was more water now in A than in C. Questioned as to the reasons for his answer the child indicated that the level in A was higher and so there was more water there. The same child questioned at a later stage replied that C contained more water giving the broadness of the beaker C as the reason for this new answer. Thus the childs thinking at this stage is of an intuitive nature based upon some limited and immediate features of the situation to which his attention has centred, leading to contradictory conclusions. The cognitive structures helping logical thinking of a deductive or inductive nature have not developed. This form of thinking which is of a transitorial nature is referred to as transductive thinking. By a series of similar experiments Piaget has shown that basic mathematical and scientific concepts such as length, area, time quantity etc. are not fully developed at this stage.

**III Concrete Operational Stage**

This is the Stage of the primary school child. By this stage the child has overcome most of the shortcomings of the earlier period and seems to have at his command a coherent and integrated cognive system with which he organizes and manipulates the world around him. These organized cognitive actions, Piaget refers to as cognitive operations. Hence the terms pre-operational for the earlier period and concrete and formal Operational for the subsequent periods: Cognitive actions pertaining to classifying setting into correspondence, ordering, distinguishing relationships, comparing, matching. etc. are all examples of such cognitive operations. Even the familiar mathematical operations of adding, subracting, multiplying, dividing, recognising equality, inequality etc. all belong to this but do not exhaust the domain of what he terms cognitive operations.

The child with this more powerful intellctual structure can now manipulate the environment in a more systematic and logical manner. Confronted with the conservation experiment the child at this stage easily arrives at the correct conclusion that the quantity of water remains unchanged on being transferred to the vessel C. He can now mentally pour back the water to B and conclude that it should result in a situation identical to the initial situation. This ability of reversibility of thought is an important feature of the thinking at this stage. So is the idea of conservation which is related to the basic mathematical idea of invariance. Thus basic mathematical concepts develop at this stage and the child can take part correctly in many logical and mathematical operations. But there are also some important limitations to his cognitive functioning at this stage. The main limitation is that all this operational thought is possible only in manipulating objects or situations that are concretely in his presence. Hence the term concrete operations. His thinking is also limited to the specific and he is unable to generalise. Though he could move a few steps ahead, he encounters difficulty in generalising or arriving at general principles. This is achieved only at the next stage of formal operations.

**IV Period of formal-Operation.**

The thinking of the secondary school child i.e. the 13 to 14 year old seems to take a different form from that of the primary school Child. Confronted with a situation the child now starts his reasoning from a more general point of view. e.g., if confronted with the conservation experiment the older child now sees it not only as a concrete situation in front of him but also as one specific instance of all the possible situations. That is, he can visualize all the possible cases of beakers getting broader and the corresponding levels of liquid getting lower and lower. His thinking process starts from the 'possible' which enables him to recognise the reality in front of him as one such situation out of the series of possible situations.

Now he can also abstract from a situation the essential variables, in this instance the area of cross section and the height of liquid. This ability to separate and abstract the variables in a situation and to manipulate them in his mind arriving at generalisations is one characteristic feature of the thinking at this stage.

The ability to visualize the possible enables him to put forward hypotheses and test them with reference to reality resulting in hypothetico-deductive thinking.

By a series of experiments with adolascent Children Piaget shows that by the end of this stage cognitive growth reaches the final form resulting in abstract thought processes characteristic of the adult.

What are the implications of these findings for curriculum reform? To consider a few:-

It helps the proper sequencing of content and grade placement of subject matter. In the existing curicula the introduction of various topics and tasks at each grade level was determined mainly by traditions They were taught in that order because it had been the way followed for years. Piaget's work indicating the capablities and limitations of the child at each stage has given a strong theoretical basis for reviewing almost all subject areas as to what should be introduced, at what level, in what form etc. In mathematics e g. it is now recognised that one should ensure that number concepts are well established before proceeding to operations with numbers. That most tasks such as ordering, sequercing, classifying, recognising relations, etc. which are basically mathematical in nature can be brought into the activities of the child at primary level in a concrete manner. That such activity forms a strong base for more abstract mathematical work at the second-day stage later. Hence the primary Mathematics programs.

Though Piaget has demarcated these age, levels, some children even at secondary school level may not have developed the cognitive structures needed for abstract thought. Even a child of 13 or 14 years may be yet operating at a concrete level. Iteuce the need even at secondary stages to start mathematics from some concrete experiences to serve as the base leading onto more abstract work next.

Based on Piagets' ideas much work has been done to develop concept maps which make clear as to what exactly should be thought at each stage. One such attempt has been by Professor Geoffrey Mathews. In this he shows how the primitive nations of sets ahd relations and the ability of representation at the pre - operational level develops into various types of varied mathematical activity at the concrete operational level, tinally leading to more abstract mathematical thinking at the formal operational level.

Earlier I referred to the idea of the structive of a discipline. Once the structive of a subject is made clear, this could be fitted on with the cognitive structure of the child at each stage thus ensuring that the child can absorb the material into his cognitive structure, resulting in more meaningful and economical learning.

4th theme

**New Findings in Learning Theory**

During the past fifty years psychologists have found much that is new about how people learn. These findings have led to new ways of looking at and arranging the teaching — learning situation.

Some such ideas which have emerged are,

1. Gestalt theories of learning.

This group of psychologists have shown that learning takes place better if the child is presented a situation as a whole and made to see its organizational principles. Once the whole situation is perceived the child could reorganize the elements into a picture which would have a more meaningful structure to him.

This would result in his learning something as a new principle which would have more lasting value in that this could be retained better and also readily applied to new situations successfully.

2. That learning takes place better if it emerges from the activity of the child. This activity may be either with concrete materials at lower stages or on a conceptual plane at higher stages. But what is important is that the child be allowed to actively manipulate the material physically, mentally or at both levels arriving at his own results. Traditional learning took an expository form where the teacher presented some idea or principle to the child in a ready made form. This type of receptive learning

often took a passive form. This was followed by exercises which were supposed to strengthen his learning and also teach him to apply these in other situations. This is a sort of teaching based on a deductive approach to learning.

In contrast the new ideas on learning emphasise the value of an inductive approach where through his own activity the child arrives at his own conclusions and generalisations. This type of leavning is also more meaningful to the child as it is his own.

3. Learning by discovery - open ended learning situations.

Traditional learning in the classroom is often limited to problem situations where the child is expected to find one correct answer. This may be good but many real problems in life are not situations which have only one right answer. There may be many or sometimes none at all! The need to lead children to more open ended type of learning has been emphasised. This type of learning where the child is encouraged to develop and manipulate the learning situation leading to his own discovery of important relationships and principles is now advocated as being of greater value. The teacher's role is more to be a guide or consultant who would be available to help the child over a difficully if the need arises or to clarify and sometimes put the child on the right track if he goes astray.

This type of leavning apart from being of much interest to the child, also motivates the child to work. It is also regarded to be of value in that he now learns how to learn which is recognized as one of the major aims of education today.

To summarise, I have touched upon some themes which have had an impact on curriculum reform.

These are,

(1) The analysis of subject content into disciplines highlighting the key ideas and concepts.

(2) The concept of the structure of a subject and the need to let the child see this structure while being engaged in learning.

(3) The ideas about the nature of cognitive growth and the value of this in organising the learning

(4) The ideas emerging out of new findings in learning theory which have led to the need for new teaching—learning situations.

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For poetry makes nothings happen; it survives.
In the valley of its saying where executives.
Would never want to tamper; it flows on south.
From ranches of isolation and the busy griefs.
Raw towns that we believe and die in; it survives,
A way of happening, a mouth.

— Auden

# Mathematics as a Social Need

*– Prof. Epasinghe*

Ladies and gentlemen! I was educated in Mathematics during a time when the so called 'New Maths' was exclusively an acadamic activity in which a few mathematicians, psychologists and educationists were involved. The number of people actively engaged in this remained small because the Russians had not sent up their Sputnik by then, and the, funding authorities of the industrial powers in the west, mainly the US, were not, at that stage, prepared to vote in money, specially by the billions, for this purely academic activity. You all know how this group was able to get financial aid by the billions for the 'New Maths' program by telling the politician that the Russians got ahead because their mathematics was modern. As a full-time student, I was not subjected to the 'New Maths' purely because I was out of place in time. Therefore I was not exposed to the so called jargon of the 'New Maths' pedlars. However, as a professional mathematician, I have subsquetly been exposed to a fair dose of 'New Maths', and therefore, I am able to use the jargon if I wish. The question I had to answer before I prepared this short talk is the following.

Should I speak in plain and simple language, my pre New Maths language, and run the risk of having an audience who would understand every word of what I say, and leave this hall at the end of this seminar by saying to themselves that the talk was nothing much because the stuff was not 'high brow'?

OR Should I clothe my talk in such a language that it will not be understood by many, so that they, will say to themselves what a fine speech it was because it was really 'high brow'.

Ladies and gentlemen, I have decided to give you the privilege of understanding what I say, and run the risk of being called a pedestrian.

I have been entrusted by the organisers of this Seminar to speak to you during the next fifteen minutes or so on the relevance of mathematics to our society. It is not my Intention to speak on its relevance to the entire society of human beings on this earth. On the contrary, I will refer only to its relevance to Sri Lanka. We are, at present, discussing what is popularly known as New Maths. The education department of the Republic of Sri Lanka has formulated its own version of New Maths for our students in grades 6, 7, 8 and 9. This is the mathematics in the new secondary school curriculum, and according to the admission requirement prevailing at present, a student will be completing his 14th year before he completes the secondary school curriculum.

What does our society in Sri Lanka demand of the secondary school leavers? What type of mathematics should they have learnt in the secondary school so that they could make a maximum contribution to satisfy the demands of our society? To what extent does the New Maths, as presently formulated, satisfy this need? These are some of the questions I would wish to answer. I do not claim an ability on my part to give perfect answers to these questions. What I can do is to state a few of my own observations in this connection. I wish to state most emphatically that I will not speak about those secondary school leavers who will reach the University and other institutes of higher education. This is because this number, as a percentage of those completing grade 9 is expected to be very small, less than 1%. Dinamına 16th Oct. 1974, G. C. E. (O. L.) No. of candidates 500,000. Dinamina 17th Oct. 1974 No. to be admitted to the University 3500.

The broad objectives of the secondary school mathematics curriculum formulated by the education department of this country are detailled in their document entitled 6 ශ්‍රේණිය ගණිතය - 1 වාරය 1972 විෂය නිර්දේශය, this being the edition which is expected to prevail at least for a couple of years. I will attempt to re-translate them to English to the best of my ability and then, they go as follows.

1. Realisation of the nature of mathematics as a versatile and widely applicable medium.

This is explained in detail and states that students should realise the following–

a. Mathematics is not a privilege only of the clever.

b. Mathematics forms a part of the knowledge essential for living in a Democratic society.

c. Fundamental concepts of mathematics are being widely used in our day to day life.

d. It is difficult to live in society without obtaining a knowledge about the activities of mathematicians, mathematical operations and mathematical method.

e. Mathematical concepts help in introducing conciseness and precision to our language in use.

f. Mathematical thought develops through the activities of ordering or arrangement in some order, quantification, relating or discovery of patterns and model building, and then this helps a student to obtain some mental satisfaction as well.

g. Mathematical languague cannot be divorced from the language in use.

2. To acquire the basic aptitudes and attitudes required for community life.

This refers to the ability expected to perform addition, subtraction, multiplication, division, day to day transactions involving arithmetic and the use of measuring devices like rulers, tapes and weighing instruments.

3. To acquire the knowledge and aptitudes required to study other subjects.

4. To develop a critical attitude required of an intelligent member of society.

5. As a foundation for further studies of fields in mathematics.

6. Understanding the mathematics requirements of the future by appreciating the history of the development of mathematics.

7. To appeciate the social role entrusted to the pure mathematician and applied mathematician.

8. To appreciate the new uses of mathematics.

9. Appreciation of mathematics for it's own sake.

As you ladies and gentlemen would whole heartedly agree, the above objectives are extremely noble and desirable. Out of these 9 objectives, which ones are important for the grade 9 leavers and which ones are only of marginal importance?

Before I answer the question, I would like to refer to a news item which appeared in the Ceylon Observer Magazine edition of the 13th of this month (in fact,only last sunday) entitled 'Agriculture popular subject in school'. Had I referred to a Lake House newspaper three years ago, the authorities would have dismissed it saying that it is a quotation from a බේරෙ ගෙදර පම පත්තරයක්. which is, in fact, what they said for the last 39 years. Thanks to the socialist program of this government, I am able to quote with authority from such papers. This news item contains the following.

Ceylon Observer Magazine Edition – 13th October, 1974.

According to a spokesman for the Education Dept. about 75% of the students in grades 6 to 9 have now selected Agriculture and its allied subjects as their pre-vocational subject.

Prevocational subject areas in relation to agriculture include agricultural science, sugar cane industry, fisheries, food crops, fruit cultivation, plantation industry, minor export and economic crops, paddy cultivation, flowers, animal husbandry and bee-keeping.

It is clear that what is relevant to 75% of the grade 9 leavers is what is important to the grade 9 leavers, and that, what out is irrelevant for those 75% would only be of marginal importance to the grade 9 leavers. Further, it is expected that most of the grade 9 leavers will end up in the youth settlements, this being consistent with the previous observation that over 75% of those in grade 6,7,8 and 9 have selected agriculture and its allied fields as their prevocational subject. The others may, if employed, go into organisations like Ceylon Transport Board as drivers, conductors and so-on or into the co-operative system as junior clerks, sales personnel and so-on or in to other fields where the mathematical skill required will be arithmetical. This leaves us no alternative but to regard only the following as important, those not mentioned being only of marginal importance, at most.

1. To acquire the basic aptitudes and attitudes required for community life.

This means numeracy that is the ability to add, subtract, multiply and divide **without the use of machines,** and the ability to use day to day measuring devices like rulers, tape measures, protractors and weighing instruments.

2. To acquire the knowledge and aptitudes required to study other subjects.

3. To develop a critical attitude of an intelligent member of society.

4. Realisation of the nature of mathematics as a versatile and widely applicable medium.

I am now in a position to make observations about the relevance of the already existing 'New Maths.' curriculum to the attainment of the above objectives.

A serious weakness in the New Maths curriculum is its preoccupation with mathematical jargon and abstract mathematical structure. This has prompted some mathematicians to define New Maths as "that branch of science over which man acquires power by giving names to things". It is, for instance, seen that, a lot of effort is being expended in trying to show the distinction between a number and a numeral.

However, it is a well established fact that it is almost impossible to say what a number is to the secondary school children in an intellectually honest and effective way. Another very obvious observation is the effort expended to show the distinction between the symbol + and — as mathematical operations on the one-hand and as adjectival prefixes on the other. From the time these symbols were invented which was several centuries ago, it was understood by even the most elementary algebra student that in the expression a + b, the sign + stands for a mathematical operation whereas in the expression + b, the sign + is only an adjectival symbol. The two concepts are so disjont that there was never the fear of confusion. But what has been done? distinction has been high lighted and even new notations have been invented. We in Sri Lanka have gone even further. Where as the word 'plus' is deemed sufficient for us. We have two words in Sinhala, ධන and ඍන ධන being the older in age. Before ඍන was invented, ධන had a meaning. Now that ධන has been replaced by ඍන and ධන is given a new meaning. Who says that, we, in this country are not creative?

The first lesson in grade 6 is on 'flow diagrams'. Since there are certain, difinite basic functions that an electronic computer can perform, any assignment entrusted to a computer for solution has to be broken up into a sequence of the basic functions the computer can perform. What happens in other situations like those considered in the first lesson in grade 6? e.g. මුද්දරයක් මිලදී ගෙන එය අලවා ලිපියක් තැපැල් කිරීම සඳහා ගැලීම් සටහනක් අඳින්න. ගැලීම් සටහන meaning a flow diagram. What are to be considered as the basic activities into a sequence of which the above activity is to be broken up? Obviously, this requires judgement, unlike in the case of the computer, where the basic activities are known. Can a 11 year old be expected to be capable of this judgement? This reminds me of the well-known encounter between the centipede and the toad. The toad had to drawa flow diagram for the movement from point A to point B by the centipede. The toad carefully observed the coordination of the legs of the centipede and constructed a flow diagram which contained instructions like 'move the left leg forward by 1 mm' and so on. Then the centipede was given the flow diagram and was requested to crawl accordingly. What was the result? A centipede in possession of a ferfect flow diagram for the activity of crawling, but unable to crawl.

It is possible to show many instances of this nature where what is emphasised within the New Maths program is not relevant to the important objectives of secondary level mathematics education, while what is not emphasised very relevant to the attainment of the important objectives.

Some of the countries in the West have designed their secondary school mathematics curricula to meet the increased demand for the use of cumputers in their society which is highly mechanised. e.g. Chapter XIII of Part I of the sixty ninth year book of the National Society for the study of education of the U.S. published in 1970 says that while 120,000 high school mathematics teachers may be required the number of computer programmers required may be about 200,000. Sri Lanka will never face such a situation in the near future when almost twice as many computer programmers will be required as high school mathematics teachers. It looks as if our New Maths curriculum has been designed for such a computer age, and may be, that it has even been designed by a computer).

Now let us look at another associated problem. Suppose we are able to draw up a curriculum which, if implemented, would deliver the goods. The success of the program will then depend on the secondary school teachers who have to do the teaching. The tale of the teacher in Sri Lanka is a very sad one. It is a very common sight to see the teacher, with the bucket of paste in one hand and a roll of handbills in the other, walking the streets and roads in his area doing what is obvious, while the educational administrator is busy having his meals 25,000 feet above sea level. Even a minor employee in the educational administration is more powerful than the teacher who is often completely at their mercy. If you were to ask a person to put down, in order of preference, the employment he would like to engage in, the position that teaching would occupy in that list would not be too far from the bottom. A teacher is one of the worst paid in the public service. A sizeable portion of his beggarly salary has to earmarked for contributions to regular poltical rallies and stunts. In this era of socialism, it is very unfortunate that the teacher, who should be one of the most respected individuals in society has instead, become one of the most repressed. I am a teacher, hailing from a family of teachers and having many friends and relations in the profession. But I have to say this that this very noble profession is being used as a dump yard for those unemployed and unfit for anything else. It is onething to scream from a political platform that the 6th grader in the most prestigous school in Colombo is being taught the same material at the same time as the child in an unheard of school in the Vanni. I would challenge them to say that the respective teachers are of the same calibre, because, it is very likely that the teacher in the Vanni is a disgruntled one who has got there due to his great virtue of having not contributed to a political rally whereas his counterpart in Colombo may be a more satisfied individual having a spouse occupying a pewerful position in the public service.

I am a member of the set in which the sovereignity of this country is vested. As such. I have a right to demand from our representatives in the National State Assembly and the administrations in education that our children should not be destroyed by subjecting them to unsuitable curricula. I have a right to demand that the secondary school curriculum in mathematics be modified by giving due emphasis to its utility value while relating the few of the desirable features. I have a right to demand that the teacher be given the place in society due to a person in one of the noblest of professions. But, to be effective I am prepared to renounce my right to demand and plead instead.

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**"Even if we resolve all matter into one kind, that kind will need explaining, and so on for ever and ever and deeper and deeper into the pit at whose bottom truth lies without ever reaching it, for the pit is bottomless"**

— O. Heaviside

# New Mathematics in Perspective

DR. R. D. STERN

I should like to examine very briefly some aspects of "New Maths" in other countries which could be relevant to Sri Lanka.

1. There seems to be no controversy that the traditional mathematics curriculum had to change. To examine why this is so, consider the following extract from the Unesco mathematics project for Arab states in 1969.

"Because Mathematics is a living growing science. Because modern society demands for more mathematical knowledge on the part of a much larger percent of the population. Because mathematics is the foundational tool of all the sciences, not only physics but the biological, earth and behavioral sciences; because the economic growth of our society depends on a far more mathematical literate population".

This sounds splendid but is also suitably vague. However to lend support to the points above consider the many topics which have recently become of general importance in Sri Lanka. Some examples are population growth, type of paddy to plant amounts of fertiliser to apply, use of crop insurance, change of crops grown due to various guaranteed price schemes or the very misunderstood topic of the standardisation of examination marks.

Quoting further from the above report on how to innovate changes in the curriculum. "To innovate new mathematics instruction is no easy task. It is of course necessary that teachers and parents of students have a fairly precise idea of the new curriculum and its objectives. One wonders whether in Sri Lanka, enough attention is being paid to keeping parents informed of the new curriculum.

2. What in general to be taught is to be taught in 'New Maths' and what are the reactions to it.

It is basically a mixture:-

a. The old syllabus eg. arithmetic, algebra, geometry.

b. Some foundation topics eg. sets and topology.

c. Some applied topics eg. statistics, use of calculating machines, computing, mathematics in industry.

Countries differ considerably in the proportions of the syllabus which are devoted to the three parts of this mixture.

In the United states, some of the courses which were introduced in the 1960's seem to have been more 'extreme' than in Britain and here (i.e. they concentrated more on the foundation topics). These courses have recently come in for very harsh critisism, in particular in the book 'Why Johnny Can't Add' by Prof. Kline, and are apparently being considerably modified.

Among parents, reaction to the introduction of 'New Maths' are fairly international. School mathematics is now a topic of conversation. On the whole parents seem largely to be opposed to the changes. They can no longer understand childrens mathematics homework. They feel that the children are not as good at arithmetic as they were and wonder where some of the new subjects are relevant to everyday life.

For a teachers reaction I should like to precis some comments from someone who has been teaching 'New Maths' in England for the last ten years. He feels that it is very good that the new curriculum has ment the end of the examination questions which asked for long involved computation. However view woefully inadequate for their need. The children are perhaps less frightened of the subject and the greater variety of topics in the syllabus certainly creates some interest. When it carries to relevance, he finds it even harder to justify to children the teaching of sets, matrices and topology than it was to justify the teaching of the various topics in the old syllabus.

3. Where do we go from here? I should like to mention three alternative which have been considered elsewhere and which might be discussed here in the future.

a. No one (thank goodness) seems to be advocating a complete return to the old syllabus.

b. In America in the 1950's (i.e. before 'New Maths') a number of books were published with such titles as 'Mathematics for Consumer' discussing the mathematics that people would make a very useful research topic as long as it is not expected to provide all the answers for a new syllabus. Two aspects that limit its usefulness are that, for example, urban and rural families may well require different uses of mathematics, also many of the uses of mathematics for adults (eg. insurance, credit schemes) are subjects in which children have very little interest.

c. The emphasis in America now, seems to be on, to some extent, changing the topics and also on changing the method of teaching away from the formal lecturing approach. This change in the method of teaching has been happening for some time at the primary level. It is now coming in to the secondary school teaching. The emphasis is to look at the mathematical aspect of real situations that are familiar and of interest to the children. There is a much greater use of games, puzzles and laboratory experiments with discs, rods, scales, levers etc. The construction of school mathematics laboratories could well be considered here.

4. Finally I should like to quote from the preface to the English S. M. P. books. This series of books has I believe had a considerable influence on the courses developed. They state "we look forward to the possibility of a more or less continuous process of change. It is to the beginnings of such a process that this series (of books) will, we hope make a useful contribution"

I hope that the courses that have so far been developed in Sri Lanka are to be seen in the same light.

## "Some Cultural Perspectives in Mathematics Down the Ages"

By Dr. V. Joseph.

*The manuscript of the above talk is not available.*

*In his speech Dr. V. Joseph surveyed the impact of Mathematics on Society since the time of the Greeks. He emphasized the Platonic tradition in Mathematics.*

*The principal aim of the seminar was to highlight the general problems that have arisen due to the introduction of the New Mathematics curriculum. Different aspects of the subject were emphasized.*

*Seminars of this nature, if held on a National basis would help to establish greater communication between professional mathematicians and those involved in the teaching of mathematics.*

# Mathematical and Astronomical Society

## ANNUAL REPORT 1974

**C. GANESHAMOORTHY** (Secretary)

I have pleasure in presenting the second annual report of the Mathematical and Astronomical Society of the Colombo Campus.

The academic year began in mid-March but due to the extended lecture hours, the time available for students' participation in the activities of the Society was curtailed. However, the untiring efforts of the Committee have been responsible for much of the achievement of the Society.

The following lectures were delivered during the year:-

1. The flight of the thunderballs (The lighting and its origin) – Dr. O. Jayaratne.
2. Statistical games to teach statistics – Dr. R. D. Stern.
3. Origin and evolution of the solar system - Dr. D. A. Mendis (University of California.)
4. ජීවිත විද්‍යාත්මක දියුණුව හා අධ්‍යාත්මික දියුණුව/පරිණාමය — ඊ. ඩබ්ලිව්. අදිකාරම්.

There were two seminars held, one on 'Teaching of statistics in Universities' and the other on 'Whither New Maths'. The participants in the former were Prof. V. K. Samaranayake, Dr. R.D. Stern, Dr. J. Kerr, Dr. R. A. Dayananda, Messrs N. P. Jeganathan, R. de Mel and P. Kathirkamanathan. We wish to express our sincere thanks to Prof. V. K. Samaranayake and Dr. R. D. Stern who were largely responsible for the success of these seminars. The seminar 'Whither New Maths' which was organized in collaboration with the Department of Mathematics was of great significance. Due prominence was given by the Press and the seminar provided an opportunity for parents, teachers and students and all interested parties to discuss this subject. Prof. V. K. Samaranayake chaired this seminar. The panel of speakers consisted of Prof. P. D. Gunatileka, Prof. P. W. Epasinghe, Dr. V. Joseph, Dr. R. D. Stern, Messrs. D. A. Perera and D. G. W. Gunapala. The discussion which followed these speeches was chaired by Prof. C. R. Kulatileke.

Films on 'Chemistry of Life', 'The Knowledge Bank and the Unexplained' were screened with the courtesy of U.S.I.S. A planetarium show on 'The Night Sky', 'Astronomical Co-ordinate System', 'Solar System and Nebulae' was held for the benefit of members who were keen on Astronomy. This was arranged by Mr. T. K. Fernando, the lecturer of the planetarium.

The Astronomical observational group of the society continued to arrange many observational evenings using the $12\frac{1}{2}$ in. telescope of the campus observatory. We thank Mr. K. V. Ratnatunge who led these classes succesfully.

Fortran Computer programming classes organised by the society were held regularly by Prof. V. K. Samaranayake. These popular classes were attended by both, our members and outsiders. We extend our sincere thanks to Prof. V. K. Samaranayake for all the effort involved.

In order to raise funds for the society, a benefit film show was organised by the society. We thank Ceylon Theatres Ltd., for lending us the film 'Cactus Flower' and we are grateful to the Management Staff of Majestic Theatre for their co-operation. We are also grateful to S. Brahmananthan (Vice President) for organizing the film show and to Miss A. Wijesinghe, Miss S. Bahawodeen, Miss S. Selvaratnam, Miss S. Navaratnam, Messrs. T. Thirukeswaran, E. N. Ferdinand, S. Somasundaram, R. Viswalingam, J. E. M. Vilvarajah, A. Senthilselvan, R. Rajakumar, M. Gopalakrishnan, Sivapragasam, and S. Srimahilkanthan for their invaluable assistance.

The Society, advised by the Professer of Mathematics initiated classes in Mathematics for the benefit of students of Development Studies of the Faculty of Education. However we were compelled to discontinue these due to lack of time.

We convey our sincere thanks to Prof. T. de S. Muttucumarana and Prof. K. D. Arulpragasam for permitting the use of the physics and Biology lecture theatres respectively and to Mr. W. R. Silva and his staff of the Physics Department for working overtime in order to keep the lecture theatres open for us. We express our thanks to Associated Newspapers Ltd, and Tim?s of Ceylon Ltd., for tbe publicity given to our lectures, seminars and film shows.

We also extend our gratitude to our staff advisers Prof. P. P. G. L. Siriwardena, Associate Prof. M. L. T. Kannangara and Dr. R. D. Stern for their invaluable advice, to Prof. V. K. Samaranayake, Dr. V.Joseph for their keen and continued interest in the activities of the society, to the clerical staff of the Department of Mathematics, especially Miss K. Velupillai, for their secretarial assistance and to Messrs. M. Thillai Nadesan and H. B. M. Munasinghe our procedessors in office for their advice.

In conclusion, we congratulate the Editorial Board of this Journal for successfully publishing another interesting issue of Sigma.

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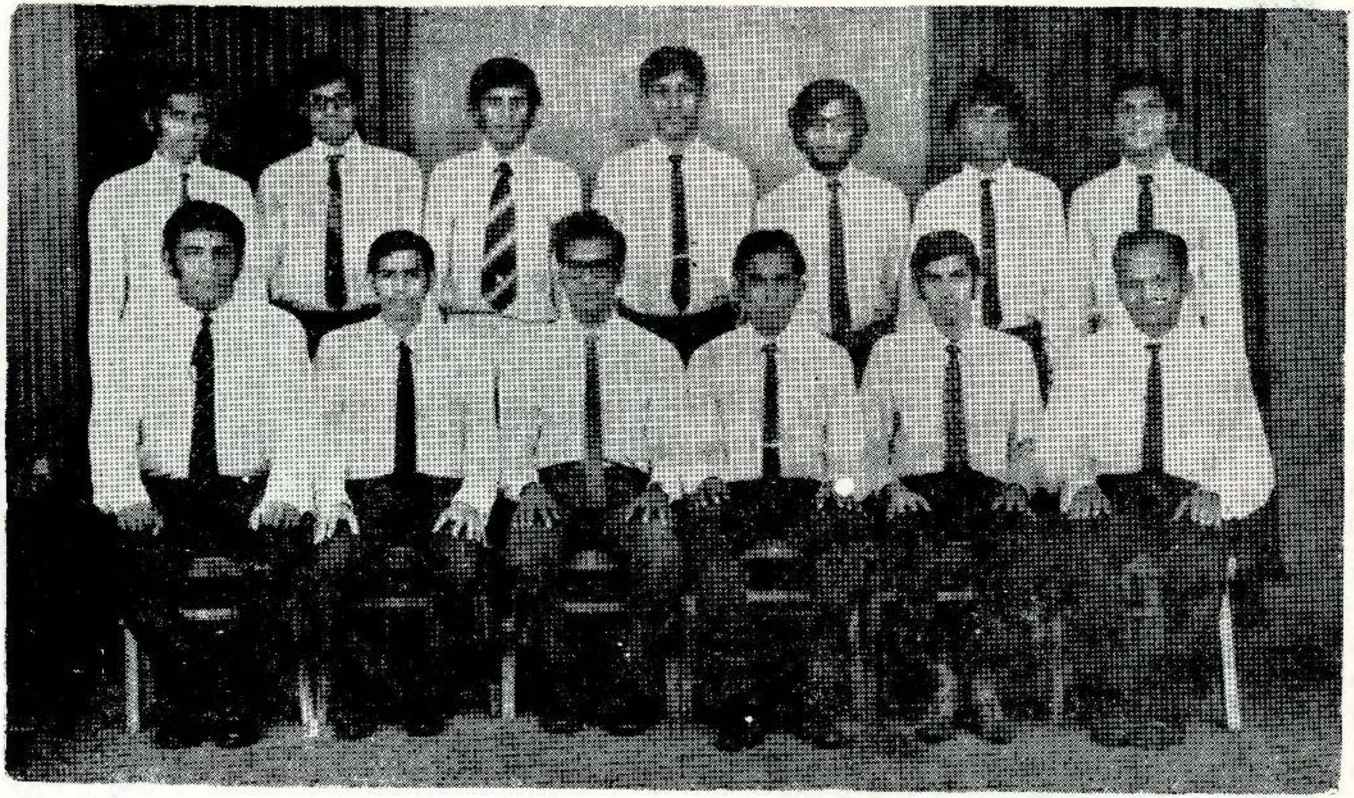

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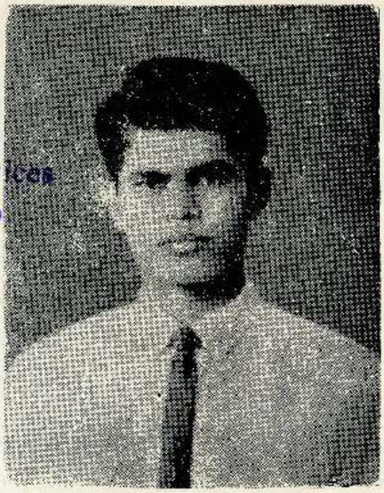

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*"A little tribute true and tender Just to say we Still Remember"*

# The Use of Numerical Results in Queueing Theory

R. D. STERN

Summary.

The first part of this article presents a very brief introduction to queueing theory. This is followed by examples of the theoretical solutions to some of the simplest queues. My claim is that, even at this level numerical examples of these formulae gives a valuable insight into their implications. If this is accepted, the use of numerical methods to evaluate the implications of some of the more complex theoretical results that are available is an important and almost totally unexplored subject. The final section presents a few of the techniques that will be involved in deriving these numerical results.

## I. Introduction

Queueing theory is a fascinating subjuct on which much work has been done in the last 25 years. To have to queue on some occasion (eg. in shops or doctors waiting rooms) is an accepted part of most peoples lives. However the range of applications of queueing theory only become apparent when we realise that it is not only people that may have to queue. Ships 'queue' outside harbours waiting for a berth to unload their cargo. Buses which break down may have to queue for someone to come to repair them. Using the last example, if in an ideal world, buses broke down at regular intervals for predictable reasons and always took the same time to repair, then the problem would reduce to one of ensuring a regular supply of men and materials, However, in the real world the time and cause of the next breakdown is uncertain, and it is this uncertainty which makes the problem much more difficult. To be more precise about the nature of these difficulties the word 'uncertainty' has to be replaced by the more technical term 'probability'. The theory of probability forms the basis for queueing theory.

Queueing theory can be studied at two different levels. At an applied level the subject is covered in all books on Operations Research or Management Science. Case studies to show the way the theory is applied to real problems are given in the books by Lee (1966), Mitchell (1972) and Martin and Denison (1971).

A vast amount of time and effort has been devoted by mathematicians to the study of queues at a more theoretical level. As early as 1957 a bibliography on the theory of queues lists 700 papers and books. However there has been little contact between the theoretical researcher and the applied scientist. Thus little of the recent theoretical work has been prompted by practical problems, and little use has been made or encouraged, of most of the theoretical results. Saaty (1966) lamented that "in the past seven years the literature on Queueing Theory has increased by half of its amount for the previous fifty years. Improvements do not match the increase in theoretical developments. Rarely has so much ingenuity been shown in tackling a variety of technical problems on paper by some of the ablest people in the world. But real life queues are still primitive. "Books on the theory of queues include these by Cox and Smith (1961), Takacs (1962), Riordan (1962), Prabhu (1965), Cohen (1969), Cooper (1972), Gnedenko and Kovalenko (1966).

The amount of work that has been done on the theory of queues can be somewhat daunting to the potential newcomer. However most of the theoretical studies have stayed within the well defined areas where elegant mathematical methods and results are available. A snmmary of the methods used and ideas for future work are given in two excellent papers by Kendall (1964) and Bhat (1969).

## 2. Some Simple Queueing Theory

To define a queueing system the way customers arrive and the way they are served has to be specified. For arrivals the interarrival distribution is considered, this is the probability distribution of the length of time between successive customers arriving to join the queue. Arrivals can occur singly or in groups. On arrival, customers wait to be served, they may be served singly or in groups. The service distribution is the probability distribution of the length of service time. The number of servers has to be specified, together with information about the order in which they serve customers. It may be first-come first-served, last-come first-served or perhaps certain types of customer have priority over others.

To illustrate the results which can be derived four examples of a queue with a single server are considered. In the first cases, customers arrive singly with unit mean intervals between successive arrivals. They are served singly, in the order in which they arrive, with a mean service time per customer of p. A basic result which is intuitively reasonable is that, if p is greater than 1, (i/e. on average customers arrive faster than they can be served) an incredibly long queue will eventually build up. If p is less than 1 the queue length and waiting time distributions for successive customers will eventually settle down to an equilibrium distribution.

Table 1. Examples of queues used in the text.

| | Distribution of Intervals between successive arrivals. | Distributionof service times | Service Regime |
|---|---|---|---|
| Example 1 | Exponential | Exponential | First come First served |
| Example 2 | Constant | Exponential | First come First served |
| Example 3 | Exponential | Constant | First come First served |
| Example 4 | Exponential | Exponential | Last come First served |

* Successive interarrival intervals are independent, with mean 1.
Successive service times are independent with mean $p < 1$.

For each of the four examples shown in Table 1, the formula for the equilibrium queue length distribution is given below. These formulae give the probability $p_j$ , $j = 0, 1, 2 \ldots$ that there are already j people in the system when a new customer arrives. These formulae are not trivial to prove, they are however some of the simplest results in queueing theory and their derivation is given in most of the standard texts on the subject.

As an illustration of the type of results to be expected, Table 2 gives the queue length distribution for the four examples when the mean service time is half the mean interval between arrivals.

Example 1 is the simplest, the queue length distribution is geometric with mean $\frac{p}{1-p}$, i.e. $P_j = (1-p)\,p^j$ , $j = 0, 1, \ldots\ldots$ (2.1)

When $p = \frac{1}{2}$ the server is idle for half the time. However, as is shown in Table 2, as many as 12·5% of the customers arrive to find three or more people in the queue ahead of them. With arrivals and service distributions of this type management has the difficult choice of having the server idle for a large proportion of the time or allowing some customers to arrive and find a long queue ahead of them. (As examples you might like to consider $p = 0{\cdot}2$ which practically eliminates the queue or $p = 0{\cdot}95$ where almost 60% of the customers arrive to find a queue of length 10 or more ahead of them)

This result is important because many practical queueing problems approximate to Example 1. However the intuitive idea that to avoid serious queues you merely have to ensure that on average customers are served faster than they arrive (i.e $p < 1$) is not true.

If the server is to work hard and at the same time long queues are to be avoided then either the way customers arrive (eg. Example 2) or the way they are served (eg. Example 3) has to be regulated (preferably both).
For example 2 the queue length distribution is also geometric $P_j = (1-\theta)\,\theta^j$ , $j = 0, 1 \ldots$, and the mean queue length is $\frac{\theta}{1-\theta}$, where $\theta$ is the smallest positive root of

$\theta = e^{-1/p}\, e^{\theta/p}$ (2.2)

When $p = 0{\cdot}5$ you can show by substitution that $\theta = 0{\cdot}203$ satisfies the equation (2.2). This is an important result; with $p = 0{\cdot}5$, by completely regulating the way customers arrive, the mean queue length is reduced to approximately a quarter of its value for Example 1.

Example 3, if $F(x) = \sum_{8=0}^{\infty} p_j\, x^j$ , then $F(x) = \dfrac{\dfrac{(1-p)(1-x)}{p\,(1-x)}}{1 - xe}$ .

The individual probabilities can be found by differentiating the generating function. The mean of this distribution is $\frac{p(2-p)}{2(1-p)}$ .

Table 2 shows that for $p = 0{\cdot}5$ the effect of having constant service times is not as impressive as having the constant intervals between arrivals of Example 2. You might like to consider other values of p in order to assess the extent to which this is a general result (i.e. if there is a choice between regulating either the way customers arrive, or the way they are served, but not both, is there always more to be gained by regulating the arrivals?)

Example 4 For this last-come first-served queue, the queue length distribution is the same as for example 1. However, for an arriving customer the distribution of the number of people already waiting in the queue is not of much importance because he automatically goes to the front of the queue. More important is the distribution of the number of arrivals before his service starts, because they will be served before him. This is given, by

$$P_e = (1 - p)$$

$$P_j = \frac{1}{2^j - 1}\binom{2n-1}{n}\frac{p^{n-1}}{(1+p)^{2n-1}}, \quad j = 1, 2........$$

One of the most satisfying aspects of queueing theory is that many of the standarp results are equivalent to results in a different branch of stochastic processes. This last result follows directly from a basic result in the theory of random walks for a simple random walk with a single absorbing barrier.

Table 2 Equilibrium queue length distribution for which the mean service time is half the mean interval between arrivals.

| | Probability n in queue before customer is served | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 or more | Mean queue length |
| Example 1 | 0 500 | 0 250 | 0.125 | 0.063 | 0.031 | 0.031 | 1.00 |
| Example 2 | 0.797 | 0.162 | 0.033 | 0.007 | 0.001 | 0.000 | 0.26 |
| Example 3 | 0.500 | 0.324 | 0.123 | 0.038 | 0.015 | 0.004 | 0.75 |
| Example 4 | 0.500 | 0.333 | 0.074 | 0 034 | 0.018 | 0.042 | 1.00 |

To see how these results could be used, consider the example of a doctors waiting room where patients have complained that on occasions they have to wait an inordinately long time before being served Suppose that the present situation corresponds approximately to Example 1. (The data collected in the investigation would of course have to be used to test the assumptions of Example 1. However in this type of study the assumption that the inter arrival and service time distributions are exponential, is often reasonable)

Two obvious solutions to the problem would be to reduce the number of patients seeing the doctor or for the doctor to reduce the length of time he is willing to see each patient. The effect of these changes can be assessed by examining the queue length distribution of Example 1 for a number of different values of p. If these solutions are not feasible another possibility might be to introduce a simple appointment system. If patients adhered perfectly to their appointments. intervals between arrivals would now be constant and the new situation would correspond to Example 2. By comparing the queue length distribution for Example 1 and 2 we can therefore try to assess whether, in this case, it is worth considering an appointment system. It must be emphasised that patients are not expected to adhere perfectly to the appointment system. However if the queue length distribution for example 2 is not very different to that of Example 1 it means that the appointment system would not be much use for perfect people and will therefore be of even less benefit for real situations i·e. to be worth considering further the appointment system must be of immense value for perfect people.

An alternative solution would be to try and persuade the doctor to reduce the variability of the service times. The extreme case when all service times are constant is given by Example 3. Finally one complaint might be that the existing system is unfair in that patients are not always served in the order in which they arrive. Example 1 and Example 4 represent the extremes of fairness and unfairness of the service regime. Thus a comparison of these distributions of the maximum amount that customers might suffer because of the unfairness of the system.

If therefore the present situation is approximately that of Example 1 in Table 2, the results indicate that the length of the queue is sometimes considerable despite the fact that the docter is free for half the time. The potential gain from having a good appointment system (Example 2) is seen to be considerable. The order in which customers are served (Compare Example 1 and 4) is in practice relatively unimportant, except perhaps as a matter of principle to the customers.

## 3. Numerical methods in queueing theory

In the last section some of the simplest queueing problems were discussed. Even for these problems the theory is not trivial and, in my view, a study of some numerical examples of the theoretical results (eg. Table 2) is helpful in evaluating the implications of the theory. Results are available for many, more complex queueing situations; however so far the work has been almost totally theoretical and numerical results are rarely given. Because of the paucity of numerical results it is often difficult to evalaute the practical significance of the theory. This is presumably one reason why the more complex theory has largely been ignored in any practical attempts to solve queueing problems. One step is bridging the gap between queuing theory and queueing practice is to translate some of the more important theoretical results into numbers.

In this section one stage in the derivation of some numerical results is described and a brief examination is made of the branches of numerical methods which this involves.

The problem considered is how to evaluate the probability distribution of the number of customers that arrive in a service time, when the intervals between arrivals have an exponential distribution. The results derived and the methods used, enable a wide range of numerical results to be given, for queues which have a variety of service distributions.

Assume therefore that the interval between arrivals has the density function $f(y) = \lambda e^{-\lambda y}$, $y > 0$. Service times have the density function given by $g(y)$, $y > 0$.

In a service time of a given length y, a standard result connecting the Poisson distribution with the sum of exponential distributions shows that

$$\text{Prob (j customers arrive in a time y)} = \frac{e^{-\lambda y}(\lambda y)^j}{j!}$$

However service times are not fixed, the overall probability of j arrivals in a service time is given by integrating over the service distribution i.e. Prob (j customers arrive in a service time),

$$r_j = \int_0^{\infty} \frac{e^{-\lambda y}(\lambda y)^j}{j!} g(y)\, dy \qquad (3.2)$$

The problem is therefore to evaluate the integral (3.2) for various functions g(y). Consider first the simple case (Example 1 in the last section) when service times have an exponential distribution.

i.e. $g(y) = \mu e^{-\mu y}$

Then
$$r_j = \int_0^{\infty} \frac{e^{-\lambda y}(\lambda y)^j \mu e^{-\mu y}}{j!} dy$$

$$= \frac{\mu \lambda^j}{(\lambda+\mu)^j} \int_0^{\infty} \frac{e^{-(\lambda+\mu)y}[(\lambda+\mu)y]^j dy}{j!} \qquad (3.3)$$

$$= \frac{\mu}{\lambda}\left(\frac{\lambda}{\lambda+\mu}\right)^{j+1} \int_0^{\infty} \frac{e^{-z} z^j}{j!} dz$$

$$= \frac{\mu}{\lambda}\left(\frac{\lambda}{\lambda+\mu}\right)^{j+1}$$

(The integral in 3.3 can easily be seen to be 1, either by integrating $\int_0^{\infty} e^{-z} z^j dz$ succesively by parts, or by looking up the Gamma function in any text on Mathematical Methods.)

Thus $r_j$, j = 0, 1 ...... is a geometric distribution. The numerical example in the last section corresponds to the case when $\mu = 2\lambda$ .

This gives $r_o = 0{\cdot}6667$

$r_1 = 0{\cdot}2222$

$r_2 = 0{\cdot}0741$

etc.

A very versatile family of continuous probability distributions has been studied by Pearson. Three of these, the Beta Type 1, (Pearson type 1), Gamma (Pearson type 3) and Beta Type 2 (Pearson Type 6), form a family in their own right and are considered here to be used as possible approximations to the service time distribution. These distributions are discussed in many texts on Probability and statistics e.g. Kendall and Stuart Volume 1 1967).

When service times have the Beta Type 1 distribution the evaluation of the integral (3.2) reduces to the problem of evaluating

$$r_j = \int_0^1 \frac{e^{-\lambda y} (\lambda y)^j}{j!} \frac{y^{\alpha-1} (1-y)^{\beta-1}}{B(\alpha, \beta)} dy \quad (3.4)$$

For the Gamma distribution

$$r_j = \int_0^\infty \frac{e^{-\lambda y} (\lambda y)^j}{j!} \frac{s^{\alpha} e^{-sy} y^{\alpha-1}}{\Gamma(\alpha-1)} \quad (3.5)$$

For the Beta type 2 distribution

$$r_j = \int_0^\infty \frac{e^{-\lambda y} (\lambda y)^j}{j!} \frac{y^{\alpha-1}}{B(\alpha, \beta)(1+y)^{\alpha+\beta}} dy \quad (3.6)$$

$B(\alpha,\beta) = \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and $\Gamma(\alpha)$ is the Gamma function

A method of evaluating each of these integrals is now considered. (i) Beta type 1.

It turns out that the integral (3.4) can be written in terms of a rather complicated sounding function, the confluent hyper geometric function.

This is denoted by M (a, b, z) and, in integral form, is given by

$$M(a, b, z) = \frac{\Gamma(b)}{\Gamma(b-a)\,\Gamma(a)} \int_0^1 e^{zt}\, t^{a-1} (1-t)^{b-a-1}\, dt$$

Hence $r_j = \frac{z^j e^{-z}}{j!} \frac{(\alpha)_j}{(\alpha+\beta)_j} M(\beta, \alpha+\beta+j, z)$,

where $(\alpha)_j = \alpha(\alpha+1) \ldots (\alpha+j-1)$, $(\alpha)_0 = 1$.

This of course has not solved the problem, it has merely rewritten it. It is however rewritten in terms of a standard mathematical function on which much work has been done (see Abramowitz and Stegun (1964). As such, tables of this function exist and hence numerical results can be given. To produce numerical results by computer a straightforward expansion of $M(\beta, \alpha+\beta+j, z)$ was used, namely

$$M(\beta, \alpha+\beta+j, z) = 1 + \frac{\beta z}{(\alpha+\beta+j)} + \frac{(\beta)_2}{(\alpha+\beta+j)_2} \frac{z^2}{2!} + \ldots\ldots \quad (3.7)$$

This formula was used to provide tables of $r_j$, $j = 0, 1, 2 \ldots$ for a variety of values of $\propto$, $\beta$, and $\lambda$ . *

(ii) Gama.

The integral 3.5 is straightforward to evaluate and gives $r_j$, $j = 0, 1 \ldots$ as a negative binomial distribution.

(iii) Beta type 2.

The integral 3.6 can be written in terms of the confluent hypergeometric function U(a, b, z) which is given in integral form as

$$U(a, b, z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

U(a, b, z) can, in turn, be written in terms of M(a, b, z) and hence the expression (3.7) can be used to evaluate $r_j$. In principle this was a very neat solution to the problem. However in practice this turned out not to be so. There seemed to be no way that the various formulae could be used to evaluate $r_j$ to the required accuracy for the variety of values of $\propto$, $\beta$ and $\lambda$ that were to be considered.

Eventually a completely different method was used. With a change of variable, $r_j$ was written in the form

$$r_j = \frac{z^j e^z}{j!\, B(\propto, \beta)} \int_0^1 e^{-z/x} x^{\beta - j - 1} (1-x)^{\propto + j - 1} dx$$

This integral was then evaluated by numerical integration. Many methods of numerical integration were tried. A method which proved satisfactory was first to evaluate where the integrand reached a maximum and then to use Gaussian integration separately on either side of the maximum.

Once satisfactory methods of evaluating these integrals had been found it proved possible to give numerical results for queues which had a wide variety of interarrival service distributions. These were used to examine how sensitive the properties of the queue were to the exact form of the service or interarrival distribution. Results could be given on the queue length and the waiting time distribution at equilbrium and on the busy periods of the queue. It was also possible to give some results on the speed at which a queue settles down to its equilibrium distribution. †

Much still remains to be done. The eventual aim of work of this kind is to apply and appreciate the practical value of more of the theoretical results that have been derived in the study of queues.

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* This worked satisfactorily except that for some of the calculations $\lambda$ was quite large. This meant that a very large number of terms in the expansion of (3 7) had to be included for the result to have the required accuracy. A variety of alternative expressions exist, ostensibly to cover this type of situation. However none of these expressions proved to be satisfactorily and hence (3.7) was always used. Further work in this field might yield a more appropriate expression.

† This is an important topic which has received little attention and on which much further work could be done.

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