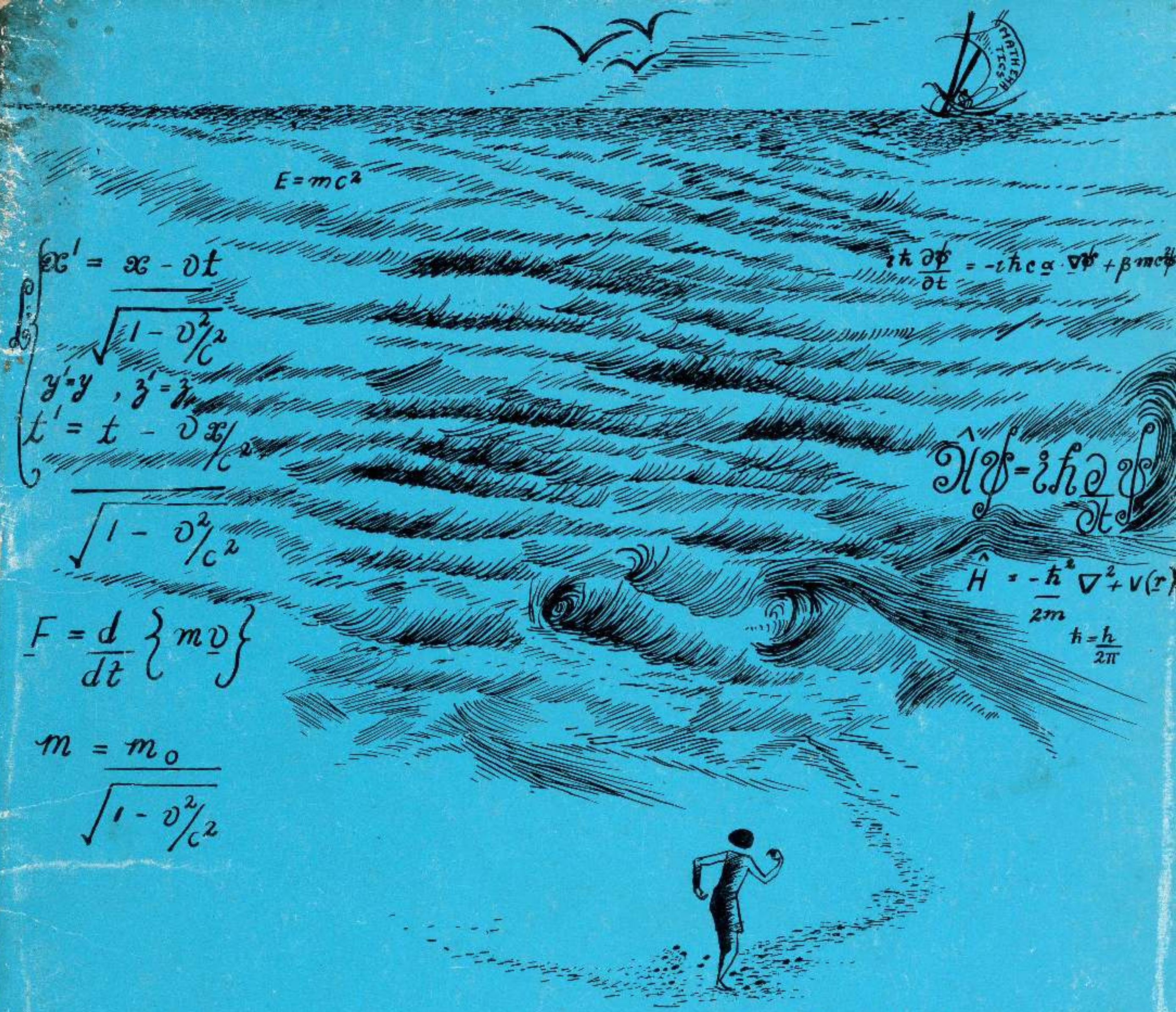

$F = \frac{G m_1 m_2}{r^3} \cdot r$

$G_i \begin{cases} t' = t, \; y' = y, \; z' = z \\ x' = x - vt \end{cases}$

$F = \frac{d}{dt}\{m v\}$

"I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the sea shore, and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary whilst the great ocean of truth lay all undiscovered before me."

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EDITORIAL

'SIGMA' has taken a new look. Previous numbers of this journal were laboriously printed out by the letter press method which involved manual type setting while this particular issue has been compiled by means of the offset process after using a word processor. The result is a cleaner clearer print, with the "printer's devil" almost exorcised!

It can be said that an electronics revolution is taking place all over the world. Computers, for instance have become essential in mass scale data processing and data storage. More and more types of work previously carried out manually is falling within the domain of computers, information technology, robots and the like. In fact, a whole new industry is being built up based on the Silicon chip.

Pause to consider the effects of the electronics revolution. While no one would deny that a machine - provided it is properly used - is quicker, more efficient, and is more accurate than a human being one wonders whether the former can always substitute the latter. A robot doctor, more accurately "a medical expert system", for instance may be able to diagnose an illness properly, but we can never expect it to provide sympathetic reasurrence to an invalid. A computer, which will patiently correct 200 mathematical problems worked out by a student cannot smile in a warm and encouraging manner with a bewildered little child on his first week at school. It is the responsibility of each of us concerned about the future of our species to see to it that this new technology is used for the benefit of humanity and not to its detriment.

ROLE OF MATHEMATICS IN PRESENT SOCIETY

Prof. P.W. Epasinghe
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It is accepted that mathematics was created by man to help him to solve certain problems which he faced in his battle for survival. Therefore, it will be correct to begin from the assertion that mathematics arose from necessity, thus giving one more example in support of the maxim that "Necessity is the mother of invention". For example, the valley of the river Nile was regularly inundated by floods, and with each flood the land markings of the fertile agricultural land got obliterated and even changed. It was therefore felt necessary to re-demarcate the boundaries of such land holdings from time to time and thus, that branch of study called geometry came into existence in Egypt. In the same way that theoretical geometry arose as an intellectual consequence of the measurement of land, it can be inferred that algebra arose as an intellectual consequence of practical arithmetic.

While battling with problems for survival, man also attempted intellectual pursuits and one of the earliest and most important problems which baffled man intellectually was 'Planetary Motion'. From the time of Ptolemy (150 A.D.), various mathematical models have been set up to explain planetary motion. However, the first breakthrough came as a result of Kepler's empirical Laws of Planetary Motion and Newton's Law of Gravitation. With the creation of the Calculus by Newton and Leibnitz towards the end of the 17th century, the problems of planetary motion got almost completely solved. A first year university student reading applied mathematics would know how to solve this problem in its most simplified form.

With the success of Newtonian mechanics in the 17th century, mathematicians and physicists began constructing more and more mathematical models to explain various physical phenomena. What our students have been learning as applied mathematics and physics in their G.C.E. Advanced Level classes and later in the universities would essentially cover most of those models. Well known examples are heat, light, sound, electromagnetism, astronomy, fluid mechanics, quantum mechanics, statistical mechanics and relativity.

Another branch of study which came into prominence in the 17th century was probability. It is believed that its invention was due to its necessity in the realm of gambling. A gambler is supposed to have questioned Galileo as to why in three successive throws of a die, a score of 10 appeared more often than 9. However, probability and thereafter statistics have become very respectable and useful branches of study. Although such studies began in the 17th century, mathematical models of situations which involved the chance element were not set up and studied so extensively as in the Physical Sciences where the models which were seriously studied were mostly deterministic and involved no chance element.

The 20th century saw a proliferation of mathematical models which involved the chance element and the most profound in the Physical Sciences was quantum mechanics. Applications of the concept of probability and statistical methods increased by leaps and bounds during this century and our universities in Sri Lanka have been conducting related courses for the last 17 years.

An examination of what we teach under probability and statistics or under titles such as operations research would show that, apart from the techniques and algorithms which may be used without understanding, the underlying mathematics is fairly sophisticated. This makes it go beyond the reach of many who have not acquired the required degree of mathematical competence. However, it is a fact that a large number of mathematical models have been set up to study and solve problems of society today, where the mathematics involved may be that of a school leaver who has been exposed to a course of elementary algebra and has acquired a certain degree of maturity thereafter. Such mathematics is now called finite mathematics, and applications are abundant in the areas of management, commerce and social sciences. With the advent of cheap pocket calculators and electronic computers the solution of mathematical problems involving large numbers of parameters and variables as well as those which involve large numbers of iterations and calculations, have come within the reach of most people.

Society today is very complex. Because of the communications revolution, different parts of the world have come closer, and organisations where millions of people interact with one-another have become very common. Before man liberated himself from tribal life, what mattered was the management of a small tribe confined to a small village. There would have been occasional wars between tribes but once the war was over, it would again have been tribal life limited to a small geographical area. With advances in civilization and the associated development of communications, the small tribal unit would have enlarged by the fusion of several such tribal units together. If the enlargement of the organisational unit is not accompanied by a corresponding improvement in the efficiency of data management and the decision making process, the unit will collapse. Whether the ancient civilizations such as the Greek and Roman or even our own of Anuradhapura and Polonnaruwa collapsed primarily due to this factor is not known, and probably will never be known with certainty. However, it seems axiomatic to assume that any organisation which involves the interaction of a large number of people who are educated in the modern sense, informed as well as misinformed by the various media which have free access to them, managed by a political process where important decisions are essentially taken by a few, will collapse unless those decision makers have efficient systems of data management and processes of objective decision making. Mathematical methods are increasingly being used in management today and the following are only a few examples.

Consider air travel. When a person wishes to get from one airport to another, there could be several different ways of doing it with varying numbers of stop-overs. For an arbitrary pair of such airports, the question of determining whether there is a direct flight from one to the other is fairly simple. However, if a person wishes to know in how many ways he can travel with one stop-over, or with two stop-overs or a still higher number of stop-overs, then the problem becomes progressively more and more difficult. With the very large number of airports in the world and the large number of routes operated by each of many airlines, the problem will be almost unmanageable except for certain special pairs of airports. Matrix multiplication which can be computerised becomes useful in solving this problem.

For example, suppose we have n airports A_1, A_2, A_3, ... A_n. We construct a matrix A whose (i,j)th element is k if there are k direct flights from A_i to A_j. The diagonal elements of this matrix are taken to be zero. If a_{ij} is the number of direct flights from airport A_i to airport A_j, then a_{ik} a_{kj} will be the total number of one stop flights from A_i to A_j with a stop-over at A_k. Therefore, $\sum_{k=1}^{n} a_{ik}a_{kj}$ will be the total number of flights connecting A_i and A_j with exactly one stop-over. But this is the (i,j)th element of A^2. Thus A^2 gives the numbers of flights with two stop-overs from one airport to another. It is now not difficult to see that A^3 gives the numbers of flights with 3 stop-overs from one airport to another. When n is small, neither matrix multiplication nor computers will be required to handle this problem. But in the world of today where n is very large, we cannot do without either of these. This same model will apply to travel by road between two cities in a country when the cities are connected by public transport systems. The same model works in an entirely different situation arising in sociology where social interactions among members of a family, for instance, are being studied. E.g. in a family of 5 people, we may construct the following matrix B.

	Father	Mother	Elder Son	Daughter	Younger Son
Father	0	1	1	0	0
Mother	1	0	0	1	1
Elder Son	0	0	0	0	0
Daughter	1	0	1	0	0
Younger Son	1	1	0	0	0

In this matrix, the rows indicate the person who consults and the columns indicate those who are consulted. Thus the (i,j)th element will be 1 if the ith person directly consults the jth person, and is 0 otherwise. It is clear that B^2 will be the matrix whose elements indicate the number of indirect influences involving one intermediary.

It is well known how the computer has changed the present day world. How a pocket calculator helps even an individual in solving problems which would have required sophisticated mathematics to solve a decade ago is described in the sequel. In the world of today, even children talk of interest rates, and both in investment and borrowing, certain compound interest calculations have to be made. Simple arithmetic says that this involves the calculations of numbers of the form $(1+r/100)^n$ where r is the percentage rate of interest for the relevant unit of time and n is the number of such units of time involved. For given values of r and n, the above value has to be obtained and the practice before calculators came was to use tables which had been constructed for specific values of r and specific values of n. Dependence on such tables will seriously limit the flexibility of lending and borrowing organisations because r and n will have to be chosen from among those tabulated in the tables. A pocket calculator will do the calculation for any reasonable values of n and r.

What about the inverse problem where for a decision process, the rate of interest is required? This means that r has to be determined from an equation which contains a number of terms of the form $(1+r/100)^n$ for different values of n. The problem reduces to the solution of equations. If the equations are of the first or second degree, then even an AL student can solve. If they are of the 3rd or4th degree, algebraic methods of solutions are within the reach of a first year university student. If the degree exceeds 4, then, no algebraic method exists for their solution. However, a pocket calculator can be used very efficiently to solve such an equation by a trial and error method, and the solutions can be obtained easily to any required degree of accuracy.

For example, suppose a person is able to invest Rs. 5,000/- in poultry farming, and estimates that, from the beginning of the 7th month onwards, he will be able to get a nett profit of Rs.400/- per month for the next 13 months and that, at the end of this period, he will be able to realise Rs.750/- by selling up. He wishes to know the rate of return on this investment so that he can compare it with other safer and more convenient investments such as in fixed deposits in state banks. If the rate of return is r% per month and a = 1+r/100, then, the equation he has to solve is,

$$f(a) \equiv 5{,}000a^{18} - 400(1 + a + a^2 + ... + a^{12}) = 750$$

which is the same as

$$f(a) = 5{,}000a^{18} - 400(a^{13} - 1)/(a - 1) = 750$$

A table such as the following may be constructed for a trial and error solution, where some arbitary value of r is selected for the first trial.

r	a	$5000a^{18}$	$400(a^{13}-1)/(a-1)$	f(a)
1.0	1.01	5980.7375	5523.732	457.01
2.0	1.02	7141.231	5872.132	1269.10
1.5	1.015	6536.703	5694.7307	841.97
1.4	1.014	6421.747	5660.0286	761.72
1.39	1.0139	6410.357	5656.5698	753.79
1.386	1.01386	6405.8065	5655.189	750.62
1.3852	1.013852	6404.8965	5654.9119	749.98
1.3853	1.013853	6405.0105	5654.9484	750.06

The first value selected is r=1 which is roughly something like 12% per annum. It is effectively 12.6825% per annum. For this value of r, f(a) is Rs. 457.01, but we want f(a) to be 750. Hence 'a' and therefore r has to be increased. We have tried r=2 next. By looking at the f(a) column, the next value of r is adjusted up or down and the entries in the table are made. The entire calculation in the table would take not more than 10 to 15 minutes and a very accurate value for r has been obtained. This value of r corresponds to an effective annual rate of about 17.95% . What we have achieved is the solution of an equation of degree 19 in about 15 minutes using a cheap pocket calculator and getting an answer for r with an error less than 10^{-4} . The error in the annual rate is less than one part in a hundred.

Another simple model is connected with inventories. An organisation which sells an item to customers cannot afford to have a situation where stocks are not available for sale because such a situation is disadvantageous to his business. On the other hand, the maintenance of large stocks is also disadvantageous because it ties down capital for long periods of time. In the alternative, if large stocks are not maintained, replenishing stocks have to be ordered more frequently, and this increases the ordering costs. It is therefore prudent for the organisation to trade off the cost of maintaining large stocks against the cost of frequent orders, and determine how often orders are to be placed and how much the size of the order should be so that the total cost is minimized.

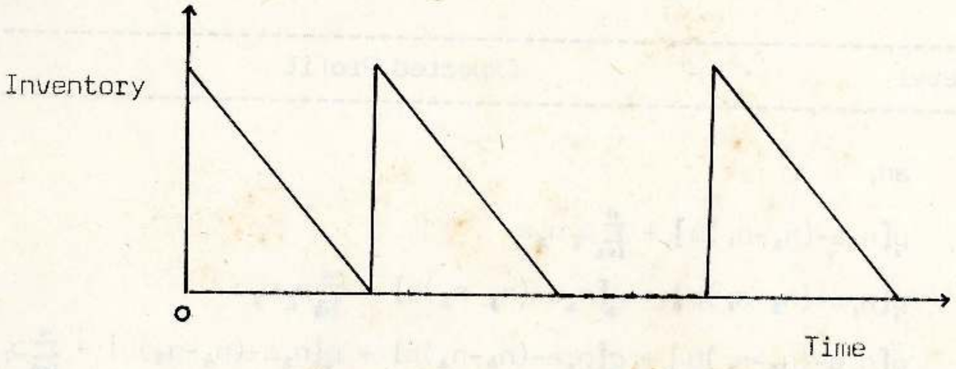
As is usually the case in mathematical models, various assumptions are made regarding the actual problem so that the corresponding mathematical problem becomes manageable. We will assume that ;

(i) stocks are depleted at a uniform rate, due to customer serving ;

(ii) there is no time difference between the placing of an order and the replenishment of the stock ; and

(iii) each order is associated with a fixed cost which would include secretarial expenses and other similar overheads, as well as transport expenses assuming that it is bulk transport and is fixed irrespective of the size of the cargo.

Let the annual demand be D and the cost of maintaining one number of the item in stock for a year be S. If the cost of each order is C, and N orders have been placed during the year, then the total ordering cost for the year is CN.



Graph of Inventory against Time

If the size of the inventory is plotted against time, the graph will be as shown. The maximum stock level is D/N which is the size of each order, and the inventory related cost which is the cost of maintaining the inventory is (1/2) DS/N for the year, the factor 1/2 arising because inventory decreases uniformly with time. The total cost is CN +(1/2)DS/N where C, D, S, are fixed and N is the variable to be determined. Costs such as the cost price of the items or the general overheads which are fixed irrespective of how often orders are placed need not be included. The simple algebraic inequality between the arithmetic mean and the geometric mean will tell us that the total cost is a minimum when the two terms are equal. This gives $N = \sqrt{[DS/2C]}$, and the optimal order size becomes $\sqrt{[2CD/S]}$.

Modification of the model when there is a time lag between the time at which an order is placed and the time when the stock gets replenished is simple. It is also possible to trade off the above cost against the loss to the organisation as a result of a stock-out situation. When the demand is not uniform but the pattern is known from past experience, a probabilistic model can be set up to determine the optimal order size. The mathematics involved in each of these cases is elementary.

Another simple model which is however probabilistic is in connection with a retailer of a perishable item such as flowers, vegetables or fresh fruit, or of an item such as a daily newspaper which loses its value after one day. Sales statistics over a period of time in the past should give the retailer the probability p that the sales level on a particular day is n_i . If these probabilities have been worked out on the records of the past N days, then $\sum_{i=1}^{N} p_i = 1$ and the mean-sales level per day will be $\sum_{i=1}^{N} p_i n_i$. If the nett profit on each item sold is 'a', and the nett loss on each item unsold is 'b', then, the expected value of the days' profit conditional on the amount that is purchased for resale can be worked out. The variation of this expected profit with the amount that is purchased for resale will enable us to pick that amount that should be purchased in order that the expected profit is maximised. For example, let us assume that $n_1, n_2, \ldots\ldots\ldots, n_N$ are in ascending order. Then the dependence of the expected profit on the purchase level will be as follows.

Purchase Level	Expected Profit
n_1	an_1
n_2	$p_1[n_1 a-(n_2-n_1)b] + \sum_{i=2}^{N} p_i n_2 a$
n_3	$p_1[n_1 a-(n_3-n_1)b] + p_2[n_2 a-(n_3-n_2)b] + \sum_{i=3}^{N} p_i n_3 a$
n_4	$p_1[n_1 a-(n_4-n_1)b] + p_2[n_2 a-(n_4-n_2)b] + p_3[n_3 a-(n_4-n_3)b] + \sum_{i=4}^{N} p_i n_4 a$
.	
.	
n_l	$\sum_{i=1}^{l-1} p_i [n_i a-(n_l-n_i)b] + \sum_{i=l}^{N} p_i n_l a$
.	
.	
n_N	$\sum_{i=1}^{N-1} p_i [n_i a-(n_l-n_i)b] + p_N n_N a$

The purchase level selected is that for which the expected profit is a greatest.

If there is a mechanism by which he is able to predict the sales level beforehand, then, there will be no losses and therefore, the expected profit in this situation in the presence of total information is $a(p_1 n_1 + p_2 n_2 + \ldots\ldots\ldots + p_N n_N)$. This will also tell him that the cost of setting up a mechanism to obtain total information about the demand on the next day should not exceed the amount by which the expected profit in the presence of total information exceeds the maximum of the expected profits.

Other techniques such as statistical methods, game theory and linear programming have been effectively used in solving problems which are relevant to day-to-day problems of the present complex society we live in. Mathematics students who are interested in using their knowledge of and experience in mathematics to enhance their career prospects should read literature on topics such as Mathematics in Management, Quantitative Decision Making and Applied Mathematics in the world of today.

EULER'S HOMOGENEOUS DIFFERENTIAL EQUATION - REVISITED

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This is an equation of the form $\sum_{r=0}^{n} a_r x^r \frac{d^r y}{dx^r} = f(x)$ where, a_0, a_1, a_2, ... , a_n, are given constants with $a \neq 0$ and $f(x)$ is a given function. The standard way of solving the above equation is by changing the variable x to t where $x = e^t$ when $x > 0$. This transforms the equation to a linear equation with constant coefficients which is then solved in the standard way. The solution when $x < 0$ can be obtained, in most of the text book cases, by mere inspection of the solution obtained for $x > 0$. Otherwise, the substitution $x = -e^t$ can be used.

An alternative method of obtaining a particular integral irrespective of whether x is positive or negative is described below. We begin with an equation of the first order and write it with the leading coefficient equal to 1, in the form

$$x\frac{dy}{dx} + py = f(x) \quad \text{where } p \text{ is a given constant.}$$

$x|x|^{p-2}$ is an integrating factor for this equation.

$$\therefore \frac{d}{dx}\left(|x|^p y \right) = x|x|^{p-2} f(x).$$

If $f(x)$ satisfies suitable regularity conditions such as those in Theorem I below at $x=0$, a particular integral of the above equation is obtained in the form,

$$y = |x|^{-p} \int_0^x u\,|u|^{p-2} f(u)\, du .$$

By changing the variable in the above integral to t where $u = tx$, one obtains

$$y = \int_0^1 t^{p-1} f(tx)\, dt \quad .... \text{(a)}$$

This result can easily be generalised to higher order differential equations of the above type. For example, the second order equation with the leading coefficient equal to 1 can be written in the form ;

$$\left(x\frac{d}{dx} + q \right)\left(x\frac{d}{dx} + p \right) y(x) = f(x) ,$$

where p, q, are given constants. This can be written as pair of first order equations in the form,

$$\left(x\frac{d}{dx} + q \right) z(x) = f(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

$$\left(x\frac{d}{dx} + p \right) y(x) = z(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

(1) may be solved using (a) to obtain a particular integral in the form $z(x) = \int_0^1 u^{q-1} f(ux)\, du$. Thereafter, a particular integral of (2) may be obtained as ,

$$y(x) = \int_0^1 v^{p-1} z(vx)\, dv$$

$$= \int_0^1 v^{p-1} \left[\int_0^1 u^{q-1} f(uvx)\, du \right] dv$$

$$= \int_0^1 \int_0^1 v^{p-1} u^{q-1} f(uvx)\, du\, dv \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (b)$$

Sufficient conditions for such a solution are discussed in theorem II below. The above solution in (b) has several advantages over the conventional method.

(1) It is a solution valid for a large class of functions $f(x)$.
(2) It can easily be generalised to an equation of any order.
(3) It is valid whether x is positive or negative.
(4) It is useful in studying the nature of particular solutions of Euler's equations.
(5) It has the beauty of simplicity.

Sufficient conditions for the above solutions

The following Theorems deal with sufficient conditions for the validity of the above solutions.

Theorem I :

Let a be a positive constant and $f(x)$ be continuous in $]0,a[$. Given p, a real constant, let $x^{p-1} f(x) \to \ell$ as $x \to +0$. Then,

$$F(x) = \int_0^1 u^{p-1} f(ux)\, du$$

is a (continuous) solution of the differential equation

$$x\frac{dF}{dx} + pF = f(x) \quad \text{when} \quad x \in]0,a[.$$

Further, $x^{p-1}F(x) \to \ell$ as $x \to +0$.

Proof:

Let $\phi(x) = x^{p-1}f(x)$, $x \in]0,a[$;
$= \ell$, $x = 0$.

Then $\phi(x)$ is continuous in $[0,a[$, and $G(x) = \frac{1}{x^p}\int_0^x \phi(t)\,dt$ is continuous and differentiable in $]0,a[$.

Further $x^p G'(x) + px^{p-1}G(x) = \phi(x)$.

$$\therefore\ x\frac{dG}{dx} + pG = f(x) \quad \text{when} \quad x \in]0,a[.$$

Further
$$x^{p-1}G(x) = \frac{1}{x}\int_0^x \phi(t)\,dt$$
$$= \frac{1}{x} \cdot x\phi(\theta x) \quad \text{where} \quad 0 < \theta < 1$$
$$\to \ell \quad \text{as} \quad x \to +0 .$$

Change of variable t of integration in the above definition of $G(x)$ to u where $t = ux$ yields,

$$G(x) = \int_0^1 u^{p-1} f(ux)\,du .$$

The theorem follows when $G(x)$ is identified with $F(x)$.

Theorem II :

Let a be a positive constant and $f(x)$ be continuous in $]0,a[$. Given real constants p, q, let $x^{r-1}f(x) \to \ell$ as $x \to +0$, where $r = \min(p,q)$. Then,

$$F(x) = \int_0^1 \int_0^1 u^{p-1} v^{q-1} f(uvx)\,du\,dv$$

is a solution of the differential equation

$$\left(x\frac{d}{dx} + p\right)\left(x\frac{d}{dx} + q\right)F = f(x) \quad \text{when} \quad x \in]0,a[.$$

Further, $x^{s-1}F(x) \to \ell\delta_{pq}$ as $x \to +0$, where $s = \max(p,q)$.

Proof :

In terms of r and s, the differential equation takes the form

$$\left(x\frac{d}{dx} + r \right)\left(x\frac{d}{dx} + s \right) F = f(x) .$$

Consider the differential equation, $\left(x\frac{d}{dx} + r \right) z(x) = f(x)$.

By theorem (I), $z(x) = \int_0^1 u^{r-1} f(ux)\, du$ is a solution such that $z(x)$ is continuous in $]0,a[$, and $x^{r-1} z(x) \to \ell$ as $x \to +0$.

$$\therefore x^{s-1} z(x) = x^{s-r} . x^{r-1} z(x)$$
$$\to \ell' \text{ as } x \to +0 ,$$

where $\ell' = \ell$ if $s = r$, $(p = q)$:
$= 0$ if $s \neq r$, $(s > r)$.

i.e. $\ell' = \ell \delta_{pq}$

Hence $\left(x\frac{d}{dx} + s \right) F(x) = z(x)$ has, as solution,

$$F(x) = \int_0^1 u^{s-1} z(ux)\, du ,$$

$$= \int_0^1 u^{s-1}\, du \int_0^1 v^{r-1} f(uvx)\, dv .$$

But $u^{s-1} v^{r-1} f(uvx) = \frac{u^{s-r}}{x^{r-1}} (uvx)^{r-1} f(uvx)$ which is a continuous function of u, v when $0 \leqslant u \leqslant 1$, $0 \leqslant v \leqslant 1$, if it is defined as $\ell \frac{u^{s-r}}{x^{r-1}}$ when $uv = 0$ and $x \in]0,a[$.

$$\therefore F(x) = \int_0^1 \int_0^1 u^{s-1} v^{r-1} f(uvx)\, du\, dv \quad \text{is a solution of}$$

$$\left(x\frac{d}{dx} + p \right)\left(x\frac{d}{dx} + q \right) F(x) = f(x)$$

where $x^{s-1} F(x) \to \ell \delta_{pq}$ as $x \to +0$.

Theorem II is thus proved.

This theorem can easily be generalised to the n-th order equation of the form,

$$\prod_{r=1}^{n} \left(x\frac{d}{dx} + p_r\right)F(x) = f(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (c)$$

where $f(x)$ is continuous in $]0,a[$, $p_1, p_2, \ldots\ldots, p_n$, are real constants and $x^{r-1}f(x) \to l$ as $x \to +0$, where $r = \min(p_1, p_2, \ldots, p_n)$.

Theorem III :

Under the above conditions,

$$F(x) = \int_0^1\int_0^1\int_0^1 \cdots \int_0^1 u_1^{p_1-1} u_2^{p_2-1} \cdots u_n^{p_n-1} f(u_1 u_2 \ldots u_n x)\, du_1 du_2 \ldots du_n,$$

where the multiple integral of order n is taken over the hypercube $0 \leqslant u_1, u_2, \ldots, u_n \leqslant 1$, is a solution of the differential equation (c) for all $x \in]0,a[$.

Further $F(x)$ is continuous (differentiable to order n) in $]0,a[$ and $x^{s-1}F(x) \to l\delta_{rs}$ as $x \to +0$, where $s = \max(p_1, p_2, \ldots, p_n)$.

Notes :

(1) The property $x^{s-1}F(x) \to l\delta_{pq}$ as $x \to +0$ proved in Theorem II is sufficient for the proof of the generalisation given in Theorem III

However, it is easily proved that $x^{r-1}F(x) \to \dfrac{l}{s-r+1}$, as $x \to +0$.

$$F(x) = \int_0^1 u^{s-1}z(ux)\, du$$

$$= \frac{1}{x^s}\int_0^x t^{s-r}\,\psi(t)\, dt \quad \text{where} \quad ux = t$$

and $$\psi(t) = t^{r-1}z(t), \quad t \neq 0;$$
$$= l \qquad , \quad t = 0.$$

$$\therefore F(x) = \frac{1}{x^s}\,\frac{x^{s-r+1}}{s-r+1}\,\psi(\theta x) \qquad \text{where } 0 < \theta < 1 .$$

$$\therefore x^{r-1}F(x) \to \frac{l}{s-r+1} \quad \text{as } x \to +0 .$$

(2) The limit $x^{s-1}F(x) \to l\delta_{rs}$ as $x \to +0$ in Theorem III too can be sharpened to read

$$x^{r-1}F(x) \to \frac{\ell}{\prod_{i=1}^{n}(p_i - r+1)} \quad \text{as} \quad x \to +0$$

and is easily established .

(3) If in Theorem III , $p_1 = p_2 = \dots = p_n = p$, the solution $F(x)$ can be expressed as a simple integral. The solution is ,

$$F(x) = \frac{(-1)^{n-1}}{(n-1)!} \int_0^1 u^{p-1} (\log_e u)^{n-1} f(ux)\, du .$$

(4) The conventional solution obtained by the change of variable $x = e^t$ can be recovered from the result in Theorem III by change of variables such that $u = u_1 u_2 \dots u_n$ and then performing n-1 of the repeated integrals .

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All the effects of nature are only the mathematical consequences of a small number of immutable laws.

– P. S. Laplace

ANALYSIS OF MULTIPLE CHOICE QUESTION PAPERS WITH SPECIAL REFERENCE TO THOSE SET AT THE G.C.E. (ADVANCED LEVEL) EXAMINATION.

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1. Introduction :

This Analysis was undertaken as an exercise to help understand the value of, as well as the problems associated with, Multiple Choice Question Examinations in measuring student ability or achievement. It formed a small but by no means insignificant part of a study to investigate the feasibility as well as advantages and disadvantages of introducing MCQ type tests for separate university admissions in Sri Lanka.

Part (I) of the G.C.E. (Advanced level) examination in the subjects botany, zoology, chemistry and physics is of the Multiple Choice type while Part (II) is of the essay type. In this study the MCQ physics paper of 1983 was analysed according to certain methods indicated later on.

2. The Structure of the Paper :

The MCQ paper consists of 60 items to be answered in 60 minutes. Each item has a stem describing the question and 5 options, one of which is the correct answer, the others being distractors. The answer sheet consists of rectangles containing labels for the 5 options (1,2,3,4,5), candidates are expected to mark the correct option. There is no correction (penalty) for guessing.

The rubric of a MCQ paper is very important, particularly when the candidates have not taken such a paper before. The rubric of the paper under investigation states as follows:

" Note:

(i) Answer all the questions.

(ii) In each of the questions 1 to 60, pick one of the alternatives (1),(2),(3),(4),(5) which you consider is correct or most appropriate.

(iii) Mark a cross (x) in pencil in the cage corresponding to the number of your choice in the answer sheet.

(iv) Further instructions are given on the back of the answer sheet. Follow them carefully.

[g=10N kg-1] "

Usually, MCQ papers are marked by making a stencil showing the correct responses. It is also possible to enter the responses into a computer and use the computer to mark as well as to produce the 'guessing corrected' score and an analysis of the test and its constituent items. If the data entry is double checked, the use of computer would not only eliminate scoring errors but also give other results useful for updating an item, particularly if it is part of an item bank.

3. Results of Item Analysis :

The responses to 60 items of a sample of those who sat the G.C.E. (A.L.) examination in physics 1983 consisting of 500 Sinhala, 500 Tamil and 101 English papers was analysed using the resources of the Colombo University Computer Centre. The analysis kept the three media separate.

In this study only one of the subjects was considered and the samples were not strictly random as our aim here is to illustrate the method of analysis rather than the actual results. Further details would be published elsewhere.

The reliability of the Test was measured using the Kuder Richardson formula 20,

$$r = \frac{K}{K-1} \left(1 - \sum_j \left(P_j (1-P_j) \right) / \sigma^2 \right)$$

where K is the number of items of the test
P_j is the proportion of correct responses to item j
and σ^2 is the variance of the test scores.

Values 0.64, 0.68 & 0.75 respectively were obtained for Sinhala, Tamil & English media indicating that the test could be further improved by either using items with higher discrimination or increasing the number of items or both.

The data available, in the form of responses to the 60 items were analysed in two different ways:

i) Classical Analysis
ii) Analysis using the Rasch model

3.1 Classical Analysis :

The first 10 columns of Table (I) gives the distribution of responses among the 5 options as percentages, together with the percent correct, wrong, multiple answers and omits for the subject physics in all media (i.e. Sinhala, Tamil & English respectively.).

Table I

ITEM		1	2	3	4	5	MULT	WR	OMIT	PBC	F.I.	D.I.
	S	25.81	24.34	15.65	16.84	16.33	0.05	82.18	0.97	0.16	0.17	0.14
1	T	20.33	27.32	17.85	17.20	15.09	0.18	80.77	2.02	0.27	0.17	0.24
	E	20.79	20.79	13.86	26.73	15.84	0.00	71.29	1.98	0.49	0.27	0.46
..	.	.....	.....	.....	.....	.....	....	.....	....	....	....	....
	S	14.87	38.78	17.57	14.89	12.94	0.11	60.38	0.84	0.25	0.39	0.25
10	T	11.04	37.63	20.98	13.89	14.90	0.28	61.09	1.29	0.35	0.38	0.37
	E	20.79	42.57	12.87	8.91	8.91	0.00	51.49	5.94	0.33	0.43	0.41
..	.	.....	.....	.....	.....	.....	....	.....	....	....	....	....
	S	8.99	24.64	15.73	40.56	8.94	0.22	83.35	0.92	0.17	0.16	0.12
60	T	8.83	20.70	14.81	43.51	10.21	0.09	83.35	1.84	0.17	0.15	0.12
	E	3.96	17.82	19.80	47.52	7.92	0.00	77.23	2.97	0.14	0.20	0.02

Although the responses to the distractors should be uniform in an ideal situation, a response pattern as found for item 10 would be quite acceptable. However, items such as:

1,7,8,9,11,12,13,14,16,17,25,27,30,31,37,38,39,45,47,49,50,51,52,54,55,56, 57 & 60 ,

were distractors (for all media) having more responses than the correct response, are not acceptable. These items can be easily improved by modifying or replacing the distractor concerned.

There were other items of a similar nature where the above situation occured for one or two media only. These items were

Sinhala Medium : 6,18,29,35,58,59.
Tamil Medium : 19,22,36,46.
English Medium : 6,18,19,29,35,36,46,58.

Column 10 of table (I) gives the percentage of omits. This figure is not high for any item . Hence it is clear that students have been guessing when they could not find a suitable answer.

The existance of three media make the task of setting a fair test more difficult. Comparison between media in table (I) show items which differ in their response patterns. These differences could be attributed to item 45 as an example among others, inconsistency in the translation.

The next step in a Classical Item Analysis is to calculate the Facility Index and the Discrimination Index. Each was calculated using two different definitions and these results appear in columns 12 & 13 of Table (I).

The Definitions used are:

Column 12. Facility Index F.I. $= c/n$

Column 13. Discrimination Index D.I. $= tc/t - bc/b$

Column 11. Point Biserial Correlation between item score (1 or 0) and score of candidate for all items.

Where n = number of candidates
c = number correct for an item
t = number of candidates in top 27% of sample
tc = number correct for an item among top 27% of sample
b = number of candidates in bottom 27% of sample
bc = number correct for an item among bottom 27% of sample

Questions 3, 11 & 55 had errors of translation into Tamil. Hence each student had been given a mark for each of these items. However results regarding these items were analysed using the original responses and in the results in Table (I), translation errors are shown by the non uniformity between media.

At this stage, we will only comment about the low facility value of items 50 & 54 which are low for all media. This suggests its modification (or rejection) before re-use in the item bank, as such items tend to lower test reliability. We would like the reader to note the differences between media, of the facility index for each item as seen on column 12.

Ebel (1979) lays down criteria for rejecting or modifying items on the basis of their Discrimination Index D. Table (II) gives these together with items of the test under analysis, that fall into the various categories.

Table II

Classification of items by Discrimination

	Medium	Items	Total
1. Very good	S	2,5,15,35.	4
items	T	2,5,15,19,33.	5
D >= .40	E	1,2,5,6,10,15,18,19,20,35,43.	11

Items 2,5,15 are common to all and can be can be classified as very good items.

	Medium	Items	Total
2. Reasonably good but possibly subject to improvement .39 > D >= .30	S	3,7,13,19,22,24,26,31,40,43.	10
	T	4,6,7,10,13,20,34,35,40,43,44.	11
	E	4,12,24,33,39,40,44.	7

Items 19,35,40,43 are common to all and can be classified as reasonably good but possibly subject to improvement.

	Medium	Items	Total
3. Marginal Items, usually needing improvement .29 > D >= .20	S	1,4,6,8,18,23,25,28,33,41,44,51,52,53.	14
	T	1,22,23,24,25,26,29,31,32,41,46,53,57.	13
	E	7,13,17,23,25,32,41,48,49,51,53,56,57, 58,59.	15

The items 1,4,6,7,13,23,24,25,33,41,44,53 are common to all and can be classified as marginal items, usually needing improvement.

	Medium	Items	Total
4. Poor items to be rejected or improved by revision D < .19	S	9,11,14,16,21,27,30,36,37,38,42,45,47, 50,54,55,60,10,12,17,20,29,32,34,39,46, 48,49,56,57,58,59.	32
	T	9,11,14,16,21,27,30,36,37,38,42,45,47, 50,54,55,60,3,8,12,17,18,28,39,48,49, 51,52,56,58,59.	31
	E	9,11,14,16,21,27,30,36,37,38,42,45,47, 50,54,55,60,3,8,22,26,28,29,31,34,46, 52.	27

If we use the Point Biserial Correlation (PBC) instead of D in Table II, then only a few items would remain as good. The items classified according to this method are given below, in Table III.

Table III

Classification by Point Biserial Correlation

	Medium	Items	Total
1. Very good items	S	19,25,35.	3
	T	2,7,15,19.	4
PBC >= .40	E	1,2,5,6,7,11,12,19,35,43,44.	11

Only item 19 is common to all and can be classified as a very good item.

	Medium	Items	Total
	S	6,29,44.	3
2. Reasonably good items.	T	5,6,10,12,13,20,25,31,33,35,40,43,44.	13
.39 > PBC >= .30	E	4,10,15,18,20,21,24,25,29,31,40.	11

Items 6,25,35,44 are common to all and can be classified as reasonably good items.

	Medium	Items	Total
	S	1,2,4,7,10,13,20,22,24,31,33,37,38, 40,41,56.	16
3. Marginal items .29 > PBC > .20	T	1,4,8,11,16,22,23,24,26,29,32,34,41, 52,57.	15
	E	8,9,13,14,22,23,37,39,41,46,47,51,52, 56,57,58.	16

Items 1,2,4,7,10,13,20,22,24,29,31,40,41 are common to all and can be classified as marginal items.

	Medium	Items	Total
	S	3,5,8,9,11,12,14,15,16,17,18,21,23,26, 27,28,30,32,34,36,39,42,43,45,46,47, 48,49,50,51,52,53,54,55,57,58,59,60.	38
4. Poor items	T	3,9,14,17,18,21,27,28,30,36,37,38,39, 42,45,47,48,49,50,51,54,55,56,58,59,60.	28
PBC < .19	E	3,16,17,26,27,28,30,32,33,34,36,38,42, 45,48,49,50,53,54,55,59,60.	22

So far we have used the number of correct responses to calculate the various parameters.

An MCQ test can be scored in these ways:

i) c

ii) c - w /(r-1)

iii) c + o /r

where c = number correct

w = number wrong

o = number omitted

so that c + w + o = k the number of items.

r is the number of options (which is 5 in the case we are considering).

At the GCE(AL) examination no correction for guessing is made and formula (i) is used. Thus it is advantageous for candidates to even blindly guess when they are not able to answer a particular item as they would not be penalised for wrong answers nor would they be given bonus points for omiting as in formula (iii).

In our opinion it does not matter much what method is used as long as the formula adopted for correction is indicateed clearly in the rubric of the question paper. If it is formula (i) blind guessing does no harm. if it is formula (ii) guessing should be intelligent guessing in that the candidate should try to eliminate at least some of the options and guess from among less than 5 answers. For formula (iii) guessing should be done only if you feel that the probability of getting it correct is greater than 1/5 as omitting it would give you 1/5 marks anyway.

The Definitions used in Table (IV)

Column 1. Item number

Column 2. Rejected on response pattern (3)

Column 3. Rejected due to media wise differences in response pattern

Column 4. Item rejected because had wrong translations (given full marks)

Column 5. Rejected by Discrimination Index (common to all media)

Column 6. Rejected by Discrimination as measured by Point Biserial correlation

Column 7. Rejected on one or more of above counts.
Note: This accounts for 37 out of the total of 60 items.

Table IV

Item	1	2	3	4	5	Reject
1	*	*				*
2		*				*
3		*	*		*	*
4		*				*
5						
6		*				*
7	*					*
8	*					*

9	*			*		
10						
11	*	*	*	*		*
12	*	*				*
13	*	*				*
14	*			*		*
15						
16	*			*		*
17	*					*
18						
19						
20						
21				*		*
22						
23						
24						
25	*					*
26						
27	*				*	*
28					*	*
29						
30	*				*	*
31	*					*
32		*				*
33						
34						
35						
36				*	*	*
37	*			*		*
38	*			*		*
39	*					
40						
41						
42				*	*	*
43						
44						
45	*			*	*	*
46						
47	*			*		*
48						
49	*					*
50	*			*	*	*
51	*					*
52	*					*
53						
54	*			*	*	*
55	*	*	*	*	*	*
56	*					*
57	*					*
58						
59						
60	*			*		*

5. Conclusions :

Conclusions arising out of this study may be summarised as follows:

a) It is desirable to computer mark the answer scripts and simultaneously conduct a classical item analysis to obtain information such as test reliability, item difficulties etc.

b) The Test Reliability of 0.64 to 0.75 (KR20) means that there is room for improvement by using better items (say with a higher Discrimination Index) and by increasing the number of items.

c) Items should be constructed with great care and pre tests held before they are used. It would be useful to insert a certain percentage (say 10%)of items for pre-testing into the test itself, so that an item bank could be built up in a few years.

d) If the test is on several distinct areas of subject matter, it is best that items in each of these areas are grouped as sub-tests. Items within a sub-test should be arranged in order of ascending difficulty.

e) The number correct score is a good measure of ability, provided the rubric clearly indicates the method of correction.

f) Small number of omits show that candidates were 'examination wise'. Documents explaing the test with model answers should be issued and training sessions held in order to make students familiar with MCQ tests.

The above conclusions do not by any means lead to the abolition of the MCQ part. On the contrary, it lead to the strengthening of both parts of the paper and a suitable combination of the scores to give a single mark for each subject.

Studies serve for delight, for ornament and for ability.

- Francis Bacon
(Essays : Of Studies)

RELATIVITY AND SPACE-TRAVEL

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The ancients believed that the earth was flat. But with land, sea and air travel this view has changed over the years. Today everyone takes for granted that the world is round.

A similar but profound change took place in the early part of this century. A twenty six year-old clerk in the Patent Office at Berne discovered that the milieu in which man lives is four dimensional, one dimensional time being welded to three-dimensional space. It is eighty years since Einstein put forth his theory of relativity, which changed the course of history. But not everyone today is aware that the world is four dimensional although the new technology is already creating such an environment. Manned space-travel, which will soon become a reality in the next century, will no doubt provide striking evidence that the world is four dimensional:-

Identical twins A and B are born on the earth on January 1st 2500. On January 1st 2520 twin A sets out on a space journey to a distant part of our universe in a space-ship. His twin brother B stays at home on the earth. The space-ship moves with a cruising speed equal to three-fifths the speed of light. When A returns to earth the age of his twin brother B is 50 years. What is the age of A?

Most people would give the ready reply "Of course, he will be 50 years old." They believe that time is an absolute or unique quantity which flows inexorably independent of space which is three dimensional.

According to the theory of relativity, where space and time are welded together into a four dimentional manifold, the answer is : "A will be 24 years old, much younger than his twin brother B ". Those who find it difficult to accept this are very much like the ancients who, for want of experience (travel), regarded the earth as flat.

The biblical span of life is 70 years. When B dies, A will be 44 years old. A will live on earth for a further period of 26 years while his twin brother B is in the grave. An interesting question presents itself "Has A lived longer than B?" Most people would reply in the affirmative. They are wrong for they are holding on tenaciously to the notion of absolute time. According to relativity the span of life is the same in each case. Space-travel does not offer a solution to the age old problem of how one could remain perpetually young.

The present belief that space is three-dimensional and time absolute, which is held by most people, will soon disappear with space travel. It reflects in many ways the egocentric nature of man. According to the theory of relativity, what is most remarkable is that one has to respect the individual experiences of TWO humans A and B. This vision may well be the first step in the search for new values, and may pave the way for a better understanding of man's role in the universe and his relationship to his fellow human beings.

I shall leave it to the reader to draw his own conclusions about the nature of human society in the twenty first century.

People who practice science, who try to learn, believe that knowledge is good. They have a sense of guilt when they do not try to acquire it. This keeps them busy ..... It seems hard to live any other way than thinking that it was better to know something than not to know it ; and that the more you know the better, provided you know it honestly.

- Robert Oppenheimer

THE ROLE OF MODERN MATHEMATICS IN QUANTUM MECHANICS - AXIOMATIC VERSUS INTUITIVE APPROACH.

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Modern quantum mechanics is a vast field and the ever-growing literature on this subject includes contributions by mathematicians, physicists, chemists and other scientists. Today, almost all of theoretical physics is quantum physics. The mathematicians' writings in quantum mechanics include axiomatic theories, and a large number of theorems which are unfamiliar to the average working scientist. The nineteenth century mathematicians and physicists had a common dialogue. Present day modern mathematics is based on symbolic logic and the traditional mathematics lacks the precision and clarity of this new mathematics. This has indeed resulted in a breakdown of communication between the mathematicians and the average scientists, physicists and chemists working on quantum theory.

The job of a mathematician is to define very precisely and technically the domain in which his results are valid, and some of his sophisticated technical assumptions may not mean anything to the more intuitive and picture-oriented physicist. Modern mathematics distinguishes three basic kinds of structures, algebraic, topological, and ordering. The mathematical structures that are used in practice are complicated combinations of those three. In quantum mechanics, often algebraic structures are used, but rigorous axiomatic theories include topological as well. It is too well known to mathematical physicists working on quantum theory, that the mathematical image of a physical system is an operator * algebra in an Inner Product Hilbert Space. The contributions from mathematicians to the development of quantum mechanics is almost full of axiomatc theories. For example the theory of lattices plays a large role in modern quantum axiomatic theories which often use the concept of Lattice Propositions as a central concept.(1) Another example is lie algebras which appear in modern quantum mechanics-the use of the operator commutator $[\hat{A},\hat{B}]=\hat{A}\hat{B}-\hat{B}\hat{A}$, for linear operators enables certain sets of such operators to be given a Lie algebraic structure. Dirac's establishment of the link between operator commutators and classical Poisson brackets may be regarded as an expression of the existence of similar Lie algebraic structures within the formalism of quantum mechanics.(1)

There is little doubt that many mathematicians derive their problems from physical contexts, but the armoury of weapons at their disposal with which they attack is almost incomprehensible-beyond the bounderies of natural intuitions of the practical oriented scientists. This is most unfortunate and the average working scientists, physicists as well as chemists are continuing using mathematical tools about which the rigorous mathematicians have serious doubts, and the mathematicians very much indeed express their doubts in a language which appears too sophisticated and incomprehensible to the average working scientists.

In theoretical physics, there is a continuous interplay of physical and mathematical arguments, and in some cases the mathematical theory may lead to formal algebraic results which are suggestive of new physical interpretations. For example Lande(2) makes an attempt to show that quantum mechanics can be deduced from classical mechanics if we combine the latter with reasonable assumptions about the measurement process and the mathematical law of combination of transition probabilities. Here a mixture of logic and mathematics is involved, and the controversy is partly about the choice of the primitive axioms on which we are to base the physical theory.

The working scientists very often take the view point of science, which says that if there is agreement between theory and experiment then it 'works'. The author would like to analyse carefully what is meant by 'works'. To illustrate this point we shall select a few examples.

It is well known to physicists that the Bohr model for the qualitative description of the hydrogen atom is ad hoc in nature. Bohr proposed that the angular momentum is quantised and his condition for quantisation is given by the expression $l = n\hbar$ (n= 1, 2, ...). This proportion was not a definite consequence of a more general theory or the result of some fundamental insight. It was invented for the specific case of the hydrogen atom. There is one fatal error in Bohr's theory: The angular momentum of the n th level as predicted by the axiomatic quantum theory is $(n-1)\hbar$ (n=1, 2, 3, ...) and not as given by Bohr. This has been confirmed by experiment. Many quantum texts have failed to point out this inconsistency! This needs to be mentioned here because more than half a century later, the Bohr model is occasionally offered as an alternative approach to those students who were very reluctant to tackle the Schrodinger formulation of quantum mechanics which needs a little more demanding mathematics! In this context, it is appropriate quoting a reminiscence of Felix Bloch: " Once at the end of a Colloquim I heard Debye saying something like: 'Schrodinger, you are not working right now on very important problems any way. Why don't you tell us sometime about the thesis of de Broglie, which seems to have attracted some attention'."

So in one of the next colloquia, Schrodinger gave a beautifully clear account of how de Broglie associated a wave with a particle, and how he could obtain the quantisation rules of Neils Bohr and Sommerfeld by demanding that an integer number of waves should be fitted along a stationary orbit. when he had finished, Debye casually remarked that this way of thinking was rather childish. As a student of Sommerfeld he had learned that to deal properly with waves, one had to have a wave equation. It sounded quite trivial and did not seem to make a great impression, but Schrodinger evidently thought a bit more of the idea afterwards.

Just a few weeks later he gave another talk in the colloquium which he started by saying : "My colleague Debye suggested that one should have a wave equation; well, I have found one".

In physics and chemistry, we are often interested in transitions from discrete states to other states - continuous band of states. When we come to the so called continuum states it is difficult to specify precisely and completely a state function. Much of the theory of transistion probabilities which deals with the above type of transitions are based on the golden rule of Fermi. The present author finds that there is considerable disagreement amongst scientists about the precise nature of the rule and how to give a respectable derivation of it. Although derivations are found in many quantum texts, the assumptions made to derive the model are not rigorously justified. Crystal field theory is widely used as a semi-quantitative theory to deal with the behaviour of magnetic ions in solids and most physicists are aware that covalency effects introduce modifications of the theory, but not so many seem to know that an ion should not have any discrete energy levels at all in a crystal field, according to the mathematicians. The mathematicians have come up with various rival forms of axiomatic theories with almost complete disregard of working scientists and it is recently that Davies(4) suggested that some of the proposed axiomatic theories may have a classical model - they are lacking in some essential feature which would make them refer to quantum mechanics as it is understood by physicists and scientists.

The phenomenon of antoionization (weakly quantized states, sometimes called unbound states) occurs in all complex atoms and molecules. The theory of antoionization contains a number of unresolved difficulties of the most fundamental kind. To the physicists, the spontaneous decay of these unstable states presents numerous experimental and conceptual problems. When one deals with a bound state one knows that at non-relatavistic energies the model presented by the quantum theory is an excellent one and the estimates for the energies of such states can be calculated within the framework of axiomatic quantum theory. However, when one comes to the study of these antoionizing states, this is no longer so. Rigorous axiomatic theory in spite of numerous attempts, has so far failed to yield a description of antoionization which is entirely satisfactory.

The outstanding problems in the theory of antoionizing states of atoms offer, a wide variety of problems in the spectral theory of sequences of unbound operators leading to the mathematical phenomenon of spectral concentration which are rather sophisticated and interesting problems in abstract functional analysis, yet the same problem can be studied also at practically all intermediate levels as : numerical solutions of ordinary differential equations; one can use Fourier analysis or complex analysis, one can study analytic continuations of the resolvent of the Hamiltonian or the Green function. Indeed the antoionizing states have been studied by the use of all these methods and more and a great deal has been achieved in understanding their behaviour, for example, it is now possible to predict the positions of the energy levels of these states with accuracy comparable to that of the experiment. Yet the fundamental difficulties in the theory of antoionization continue to remain unresolved. (The author is not aware if this phenomenon has now been resolved completely and precisely.)

We conclude this paper with a very brief dicussion on one of the more surprising and baffling features of quantum mechanics: A composite system is incomparably more than the sum of its parts. For most conventional quantum physicists this is known as the 'Paradox Correlation'. The most disturbing consequences are philosophical in nature - quantum mechanics has effected all scientific and even general human thinking. Conventional quantum mechanics clearly and unambiguously predicts correlations. Experimental evidence have so far confirmed the correlations. Yet it is so puzzling that we are unable to understand this phenomenon which certainly does not appeal to our natural intuitions. Most physicists and chemists are aware of the classical formulation: If two systems A and B have been in contact for an indefinite period before and no matter how far apart they may be now - observing one of the systems drastically alters our view of the other in ways which are very difficult to understand.

One of the basic rules in quantum mechanics relating the combination of the two quantum physical systems A and B says: If the system A is described by the Hilbert space H_1 and the system B is described by the Hilbert space H_2 then the composite system is represented by the direct-product space $H_1 \otimes H_2$ and there are observables in the composite system that are incompatible with all observables of either system A or B. Thus, in quantum physics there exists properties that cannot be simply obtained as combinations of the properties of either subsystem - the whole is not the sum of the parts. Let us hope that one day this riddle will be solved.

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The fly sat upon the axle-tree of the chariot-wheel and said : 'What a dust do I raise.'

- Francis Bacon

THE STRONG INTERACTION

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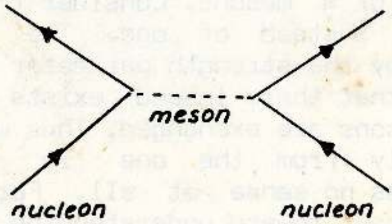
When physicists first considered the structure of the atomic nucleus, they confronted a mystery. They knew that the nucleus was composed of nucleons - protons and neutrons - and that heavy elements have many protons in their respective nuclei (Uranium for example, has ninety-two). But how can such a large quantity of protons be reconciled with the laws of electrodynamics, which prohibit the existence of so many particles of like charge in such a tiny space? The electric repulsion between these protons is so great that the nucleus should explode. According to the laws of electrodynamics, therefore, atomic nuclei should not be stable. There is only one solution to the problem: there must exist in nature another basic force that keeps the atomic nucleus together. This force indeed exists and is called the strong nucear force or the strong interaction. It is very strong indeed: a naive estimate is that the strong interaction is at least one hundred times stronger than the electromagnetic interaction.

Of course we cannot help but ask at once why we do not observe the effects of the strong interaction in everyday life the way we observe electromagnetic effects. The answer lies in a special property of the strong interaction: it is active only at short distances, distances in the order of 10^{-13} cm. As soon as nucleons are removed further apart from each other than 10^{-13}cm, the strong interaction has scarcely any effect. The dominant force between nucleons at such distances is the electromagnetic force. However, if the nucleons are brought close to each other again, the strong interaction rapidly dominates. This is why atomic nuclei are stable and do not explode. Nucleons inside a nucleus are indeed very close to each other (10^{-13} cm. or less), and their behaviour is dictated by the strong nuclear force, which dominates over the electric repulsion between the protons.

The strong interaction discriminates between nucleons and the electrons in the atomic clouds. The electrons are not affected by the strong interaction at all. The only interaction of relevance to the atomic cloud is electromagnetic interaction.

Although we cannot see the effects of the strong interaction directly in everyday life, we can at least observe them indirectly. For example the interaction is exceedingly important in nuclear power plants; the energy produced by a nuclear reactor is obtained through the rearrangement of nucleons. The same principle applies to the atomic and hydrogen bombs.

The interaction between charged objects (electric attraction or repulsion) is governed by the exchange of "virtual proton quanta between them. The question immediately comes to mind as to whether we can understand the strong interaction between strongly interacting particles by means of a similar exchange principle. The suggestion that we may do so was made about fifty years ago by the Japanese physicist Hideki Yukawa, who proposed that the strong interaction results from the exchange of quanta, called mesons, between nucleons.



It is a characteristic of interactions caused by the exchange of virtual quanta that the range of interaction is intimately related to the rest mass of the quanta exchanged. Photons, for example, are massless, and so the range of electromagnatic interaction for charged particles is infinite. This means that electric forces can be felt even at relatively large distances. There is no typical distance for the electromagnetic interaction as there is for the strong interaction. For example, we cannot say that electric forces can be neglected if we are more than 10 m. away from a charged object, but we can say that the strong interaction is irrelevant once we are more than 10^{-13} cm. away from a strongly interacting particle, such as proton. Using this property of the strong interaction, Yukawa determined the mass of the hypothetical mesons that were supposed to produce the strong interaction. He came up with a mass of about 100 MeV., that is, about one tenth the mass of the proton.

It took more than fifteen years for the existence of mesons to be firmly established by experiment. They are called π mesons (or pions) and have a mass of 140 MeV. There are three of them : π^+ , π^- and π^o the (which is electrically neutral). The π^- particle is the antiparticle of the π^+ ; the π^o particle is its own antiparticle.

Except for the obvious difference that π mesons have mass and photons are massless, the introduction of mesons as the quanta of the strong interaction allows us to develop a theory of the strong interaction that is quite similar to the theory of electrodynamics. Still, we at once encounter one serious difficulty. Suppose we describe the strength of the strong interaction in a manner analogous to the fine-structure constant α . It turns out that this constant is rather large-on the order of 10. Unfortunately, ascribing such a large value to the strong interaction strength parameter makes it impossible to develop a real theory of the strong interaction. The argument goes like this. Two electrons interact via the exchange of one virtual photon quantum between them. It is possible, though, for two photons to be exchanged instead of one, but the probability of this happening is very small, less than 1 in 10,000. Therefore we can neglect the two photons process unless we need to be utterly precise. This method of calculation occurs in perturbation theory.

The smallness of the fine-structure constant allows us to make very precise calculations in quantum electrodynamics. In the case of the strong interaction, however, it makes little sense to use perturbation theory since the corresponding strength parameter is greate than unity (10 in our case). Let us look at the interaction between two nucleons as described by the exchange of π mesons. Consider the possibility that two π mesons are exchanged instead of one. The probability of such an occurrence is determined by the strength parameter squared, which in our case is 100. This means that there indeed exists a high probability that two rather than one mesons are exchanged. Thus we encounter a situation that differs considerably from the one in electrodynamics ; and perturbation theory makes no sense at all. For this reason, little progress was made in the past toward understanding the strong interaction. One may, of course, think that this failure is simply due to the fact that we do not know how to make the right calculations in the Yukawa theory (no one, so far, has found a method besides perturbation theory to perform the calculations). It appears, though, that this is not the reason for our lack of progress. In recent years we have learned much that is new about the strong interaction.

The study of the behaviour of nucleons under highly unusual circumstances (for example, in high-energy collisions with electrons) has provided invaluable insight into the dynamics of the strong interaction. For about the past eleven years physicists have been working on a special theory of strong interaction, called quantum chromodynamics (QCD), and for the first time in the history of particle physics there is hope of understanding all strong interaction phenomena within a theory remarkably similar to Maxwell's theory of electrodynamics.

A scientist worthy of the name, above all a mathematician, experiences in his work the same impression as an artist ; his pleasure is as great and of the same nature.

\- Henri Poincare

ANALOGOUS OR SIMILAR PROOFS

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I will present here the proofs of two results in Set theory in order to observe some similarities between the two proofs. The results and their proofs are due to the same person George Cantor, the founder of Set theory. I will also present Richard's paradox which has a strong resemblance to one of the two proofs.

Result I :

The set of all real numbers between 0 and 1 is not countable (i.e. there isn't a bijection from the set of natural numbers to this set).

Proof :

Suppose f is a bijection from the set of all natural numbers to A, where A is the set of all real numbers between 0 and 1. Every real number between 0 and 1 can be written uniquely as a non-terminating decimal (e.g. 1/2 =0.4999.......). So let us represent these numbers in this way. Now consider the following number c which lies between 0 and 1. c is defined in the following manner. Let the n th decimal place of c be 1 if the n th decimal place of f(n) is 9 or 8 and let it be f(n) plus one otherwise. Then c differs from every number f(n) (since for any n, the nth decimal place of f(n) differs from the n th decimal place of c). Thus if f can't be a bijection from the set of all natural numbers to A.

The method of this proof has been called Cantor's "diagonal" procedure.

$$\begin{array}{ll} f(1) & a_{11}\ a_{12}\ a_{13} \ldots\ldots\ldots\ldots \\ f(2) & a_{21}\ a_{22}\ a_{23} \ldots\ldots\ldots\ldots \\ f(3) & a_{31}\ a_{32}\ a_{33} \ldots\ldots\ldots\ldots \\ \cdot & \\ \cdot & \\ f(u) & a_{n1}\ a_{n2}\ a_{n3} \ldots\ a_{nn} \ldots \\ \cdot & \end{array}$$

If $c = b_1 b_2 b_3 \ldots\ldots\ldots$, then for any n $b_n \neq a_{nn}$

Result II :

Any set cannot be put into one to one correspondence with it's power set (i.e. for any set X there isn't a bijection from X to P(X)).

Proof :

Suppose X is a set and f a bijection from X to P(X). Let A = $\{x \mid x \in X \text{ and } x \notin f(x)\}$. Then A $\in$ P(X) but there isn't any member t of A

such that f(t) = A. i.e. A differs from every set f(x).
(Since: Suppose A = f(t). Then either $t \in A$ or $t \notin A$. If $t \in A$ then since A = f(t), $t \in f(t)$. But $A = \{x \mid x \in X \text{ and } x \notin f(x)\}$. Therefore $t \notin A$. On the other hand if $t \notin A$ then $t \notin f(t)$ and therefore $t \in A$.) Thus we have a contradiction and hence there isn't a bijection from X to P(X).

Now let us look at some similarities between the two proofs.

(1) In both proofs the non-existence of a bijection is proved by assuming there is a bijection and then defining an element that does not appear in the range of the bijection (i.e. c in proof (I) and A in proof (II)).

(2) How is this element that doesn't appear in the range defined? In proof (I) it is defined by the "diagonal" procedure. In proof (II) too we have a "diagonal" procedure. This is better seen when we consider the special case where X is the set of all natural numbers. Let us visualize the proof in the following manner:

f(1)   1, 2, 3, 4, ...

f(2)   1, 2, 3, 4, ...

f(3)   1, 2, 3, 4, ...

.
.
.

f(n)   1, 2, 3, 4,...n,...........

Think of the above diagram this way. For the 1, 2, 3, ... written against f(1) only some of the numbers will be elements of the set f(1) etc. Now if $n \in f(n)$, $n \notin A$ and if $n \notin f(n)$ then $n \in A$. In this way A is different from every f(n).

(3) In both proofs we assume there is a bijection and then in terms of this bijection we define an element that should be in the range of f. But we obtain a contradiction with this assumption and so we conclude that there isn't a bijection. Now in this type of definition an element is defined in terms of a set of which this element is a member and also the existence of this element depends on this set. So there is an element of self-reference. We will see more of this "self reference" when we come to Richard's Paradox. However, in both these proofs we can avoid the contradiction by taking f to be just one to one only and since the element we define is not in the range of f, f is not onto the required set and hence f cannot be a bijection.

Now let us turn to Richard's Paradox.

Let E be the set of all real numbers between 0 and 1 that can be defined in a finite number of words. We can enumerate all the phrases in English which describe members of E by ordering them lexicographically. (i.e. for instance the phrase 'point six' comes before 'point three' since the numbers of letters in 'point six' is less. Also 'point six' comes before 'point two' since although the number of letters are the same when we consider the first non-identical letters 's' and 't', s comes before t in the alphabet.) This induces an enumeration of the members of E. In other words there is a bijection f from the set of natural numbers to E. Now consider the number c which lies between 0 and 1 and is defined in the following manner. The n th decimal place of c is 1 if the n th decimal place of f(n) (i.e. the n th decimal place of the n th element of ennmeration) is 8 or 9 and it is the n th decimal place of f(n) plus one otherwise. Now this number c is not a member of E but it can be described in a finite number of words, i.e. c is a member of E but it isn't equal to any of the f(n) s. This is a contradiction. However as in the above two proofs we cannot avoid this contradiction by saying that this bijection doesn't exist. So we have a paradox.

I will conclude by giving another example of self-reference that leads to a contradiction, viz the barber problem. Suppose there is in a certain town a barber who shaves all those and only those who do not shave themselves. The barber is also a person in town and he either shaves himself or does not shave himself. If he shaves himself then the barber does not shave him. But he is the barber. So he does not shave himself - i.e. we get that if the barber shaves himself then he does not shave himself. Therefore it must be that he does not shave himself. But then the barber will shave him - i.e. he will shave himself. This is a contradiction. So there can't be a barber like this.

Nature has shown over and over again that the kinds of truth which underlie nature transcend the most powerful minds.

- Subrahmanyan Chandrasekhar
(Astrophysicist, recipient of the Nobel prize for physics - 1983)

MATHEMATICAL THINKING

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The study of mathematics may be approached with a variety of objectives, and it may be looked at from equally different perspectives. Mathematical thinking is almost synonymous with systematic thinking, and mathematics forms the basis of nearly everything we do in a systematic way. We would stress the logical aspects of mathematics, presenting it as a study of abstract systems. i.e. as the study of 'games' which are played with abstract objectives whose behaviour is characterized by given sets of rules.

Whenever we desire to solve a problem systematically we must consciously or unconsciously analyse the abstract objects with which we have to deal and the rules which they obey. By first translating the problems into the language of mathematics we not only eliminate irrelevant detail, but we put ourselves in a position where the solution of the problem will become a mechanical routine. The idea of identifying real objects with abstract mathematical objects is of particular importance in theoretical science, e.g. in atomic physics. It is here that the scientists try to construct abstract mathematical systems whose elements can be identified with electrons, protons, neutrons and other objects which nobody has ever seen. If it can be shown that the behaviour of certain mathematical objects corresponds to the indirectly observed behaviour of electrons,neutrons,etc, the mathematical objects can then be used as the abstract counterparts of these physical objects. Of course, all this is easier said than done, because there is always the question whether we are working with the right kind of a mathematical system. i.e. whether the real objects with which we are concerned actually do constitute a permissible interpretation. It should be understood therefore, that if theoretical scientist identifies an electron with a complex wave function he is, logically speaking, doing the same thing which we do when we identify a small heap of chalk with a mathematical object called a point. The problem of finding a suitable mathematical model, namely, suitable mathematical objects with which real objects or concepts may be identified is of major importance in all branches of science.

Although much progress in mathematics results from the demands of scientists and others who need abstract mathematical systems in their work, many mathematicians choose their profession because of the enjoyment and the challenge which it provides. They find pleasure in studying systems of abstract objects without worrying or caring too much about the possible interpretations. If they tire of one mathematical system they have only to change a few of the postulates, and they are faced with an abundance of new and challenging problems. Perhaps this may sound frivolous, but we should add that most of the systems which have been developed by mathematicians have sooner or later found fruitful and sometimes surprising applications. To give examples we have only to look at complex numbers and their applications in physics and engineering or at Boolean algebra and its application to the analysis of electric circuits.

In a world so full of evil and suffering, retirement into the cloisters of contemplation, to the enjoyment of delights which, however noble, must always be for the few only, cannot but appear as a somewhat selfish refusal to share the burden imposed upon others by accidents in which justice plays no part. Have any us the right, we ask, to withdraw from present evils, to leave our fellow-men unaided, while we live a life which, though arduous and austere, is yet plainly good in its own nature?

- Bertrand Russell

GEOMETRICAL REPRESENTATION OF A GEOMETRIC SERIES

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Introduction -

It can be easily proved by elementary algebra that the sum of the first n terms of a geometric series (which can be represented in the usual notation by $a + ar + ar^2 + \ldots\ldots\ldots + ar^{n-1}$) is given by $a(1 - r^n)/(1 - r)$. It is shown here that for some special cases the above result can be obtained by geometric means as well.

The following notation is used throughout ;

R - the set of real numbers
N* - the set of natural numbers
$A = \{ x \,/\, x \in R \wedge x > 0 \}$
$B = N^* \setminus \{1\}$

$a \in A,\ r \in B$

Consider a closed curve of arbitrary shape. The area bound by this curve can be expressed in the form ar^n, $a \in A$, $r \in B$. Divide this into r parts of equal area, each being ar^{n-1} units.

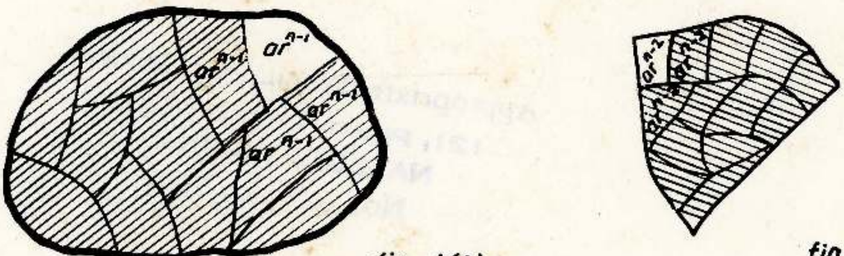


fig. 1 (a) fig. 1 (b)

Shade $r - 1$ parts. Thus the total shaded area is $(r - 1)ar^{n-1}$ [fig 1.(a)]. Divide the unshaded section into r equal parts each of area ar^{n-1} [fig 1.(b)]. This increases the total shaded area to $(r - 1)ar^{n-1} + (r - 1)ar^{n-2}$ units. Continue this procedure. After the nth time, there will remain an unshaded area of 'a'.

Thus, the total shaded area is given by $ar^n - a$ as well as by $(r - 1)ar^{n-1} + (r - 1)ar^{n-2} + \ldots + a(r - 1)$.

i.e. $(a + ar + ar^2 + \ldots\ldots\ldots\ldots + ar^{n-1})(r - 1) = a(r^n - 1)$

$a + ar + ar^2 + \ldots\ldots\ldots\ldots + ar^{n-1} = a(r^n - 1)/(r - 1)$

Hence the formula is true when $r \in N^* \setminus \{1\}$.

$a \in A$, $r \in \{p \,/\, p=1/n \wedge n \in B\}$.

Take a closed curve of arbitrary shape, enclosing an area 'a', say. Divide this into x equal sections (here $x \in B$), shade $(x - 1)$ parts [fig 2] and partition the unshaded area into x sections of equal area. Repeat this process upto the nth time. The final shaded area equals $(x - 1)a/x + (x - 1)a/x^2 + .. + (x - 1)a/x^n$. i.e. $a - a/x^n$ units.

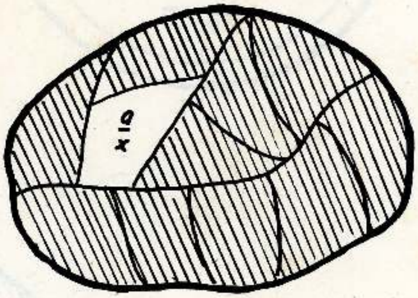


fig. 2

$$(x - 1)\,(a/x + a/x^2 + \ldots + a/x^n) = a(1 - 1/x^n)$$

$$[(x - 1)/x]\,(a + a/x + \ldots + a/x^{n-1}) = a(1 - 1/x^n)$$

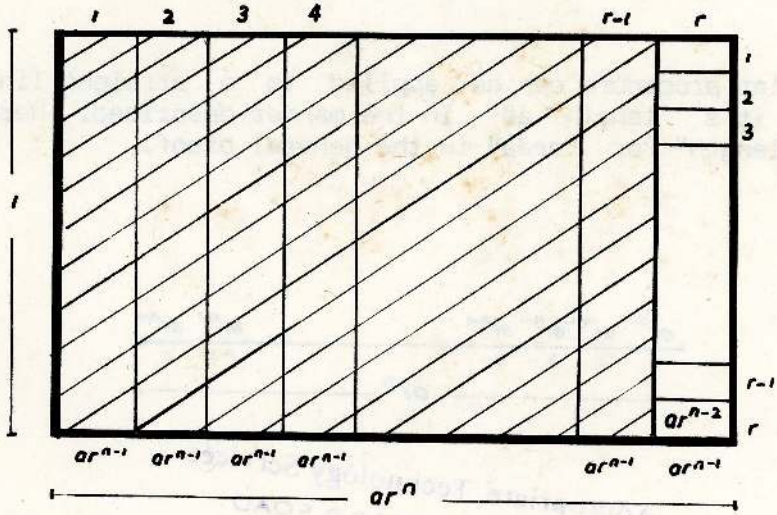
$$a + a/x + \ldots + a/x^{n-1} = [a(1 - 1/x)] \,/\, (1 - 1/x)$$

Taking $r = 1/x$, (then since $x \in N^* \setminus \{1\}$ and is finite, $0<r<1$)

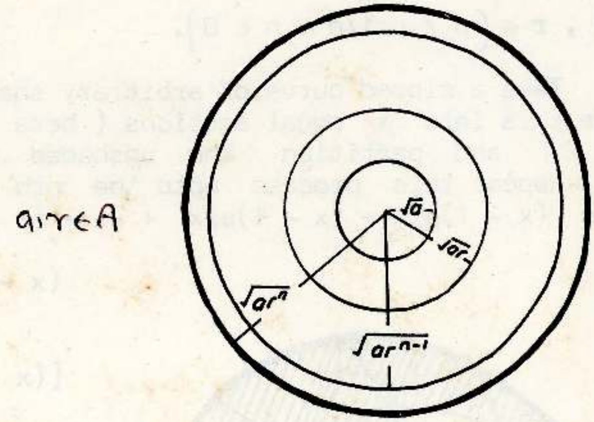
$$a + ar + ar^2 + \ldots + ar^{n-1} = [a\,(1 - r^n)] \,/\, (1 - r)$$

Deriving the formula from some familiar geometrical figures.

Consider the closed curves that take the form of (a) a rectangle (b) a circle (c) a triangle. Since the relevant procedure has been outlined for an arbitrary closed curve, the diagrams are self explanatory.



(a)



(b)

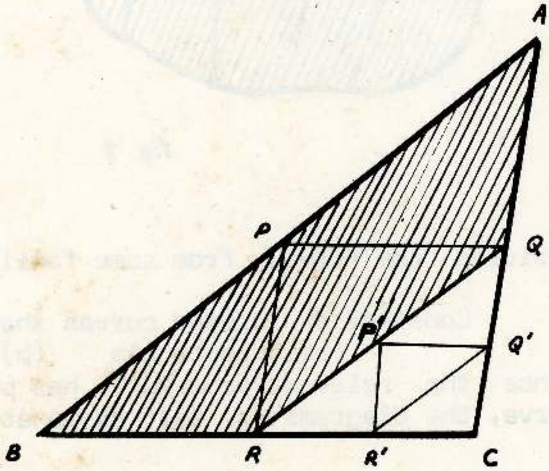
Δ ABC $= a = 1$ $\quad x = 4$

AP = PB = BR = RC = CQ = QA

QP′ = P′R = RR′ = R′C = CQ′ = Q′Q

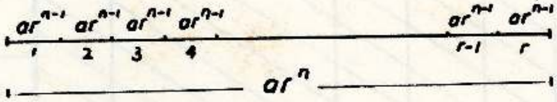
. . . .

. . . .



(c)

A similar procedure can be applied to a straight line segment, dividing up it's length ar^n in the manner described. Here we must substitute "length" for "area" in the general proof.



AMAZING "CREATURES" OF THE MODERN WORLD

Haresh Siriwardene
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As citizens of Sri Lanka, we can be proud of the developments in electronics that are taking place today in our country. The opening of the Arthur C. Clarke centre was a great step forward in this direction. Yet we are far behind the technological advances that are taking place in countries like the United States, Japan, etc.

One of the main objectives of modern technology is to reduce the drudgery of human labour. People, in general, do not like to exert themselves too much mentally or physically. But no machine that man has invented can match the capacity for creative thinking that the human brain possesses. Physical exhaustion, however can be reduced by the use of mechanical devices. One of the new labour saving devices used extensively throughout many industrial countries is the robot.

What is a robot? The word robot brings a picture of a 'steel man' to our minds. According to an earlier definition the word 'robot' was explained as 'a creation similar to a human being'. But with the actual manufacture of robots the meaning of the word too has changed. According to the new definition, 'robots can work side by side with humans, and also can be controlled by human made computers'.The definition which implies that a robot should look like a human can now be discarded; one has to keep in mind that a robot need not be similar in appearance to a human being. These amazing "creatures" can perform a variety of activities. If we want to get various things done by a robot, then for each and every such activity we should program the robot.

The concept of the robot arose far back in ancient times. In 400 - 350 B.C. the Greeks constructed a wooden pigeon that could fly. In the history of Alexandria there is mention of a group of dancing robots. A scientist named 'Sir Albert' invented, in the 13th century, a robot (using glass, leather and metal) to welcome his visitors. In Switzerland, Paracells in his lectures on 'How to Make an Artificial Human' mentioned a list of requirements which included human blood and the organs of a male human being. In the 16th century, a man named Praglored had made a huge robot out of clay. In 1942, a roboist stated that 'Robots won't harm humans, but they do always obey human orders, and they will safeguard humans'.

In the present day, many tremendous changes are taking place in the use of robots. For the past two decades, industrialists in Japan, America and Europe have got most of their routine and tedious work done by robots. The latest creation in robots was produced in Japan, and named the 'Robot Nurse'. The impersonal nurse with hard metal hands does her work according to the clock. In some factories in Japan it is not possible to find a human being for miles. But still the work is done neatly and smoothly by the efficient hands of robots. At the end of 1981, statistics showed the robot population in the world was about ten thousand. About 5,000 of them are in Japan and are used in the motor car industry. In the United States there are about 4,000 robots, while in Europe there are about one thousand. The new generation robots are small in size and seem to be freely available. These small sized robots are named 'Puma' and belong to the second generation of robots. These robots not only have metal hands but also a computerized brain. Therefore each and every event could be programmed with precision. Each Puma is worth about 20,000 sterling pounds.

You can realize how much time you and your family could save if you own a device which could cook your meals, clean the house and garden and also wash your clothes. There are many more important things we could do with the help of robots. For example, let us consider the painting of a motor car. Here the spraying has to be done very carefully and systematically, otherwise the result would be crude and ugly. Therefore the painting robot should follow certain steps, and its hands have to be controlled from the start to the end. Its brain has to be programmed in such a way that its hands are attracted to where the paint is required. Also if once a specific place is painted a signal has to go to the brain, so that its hands could move to the next spot to be painted. The hands of the robot have to move in one direction. This method is called the 'Random Centrifugal Memory Method'. In this method, there will be two limits drawn out for the robot.

(a) The robot has to learn while working.
(b) It should know where its hands are placed.

At present, the qualities and abilities of robots are being constantly improved. Sixty percent of the developments carried out are related to the motor car industry. A professor from the University of Nottingham has invented a robot that can see. In this case the robot will be much more efficient as loading, unloading, painting and fixing the spare parts can be done easily.

There has been a murder case recorded in the world of robots. In a factory in Japan where both robots and men were working, the robot had come very close to a man, raised its hands and strangled him. The man could have survived if he had only known that a robot breaks down when the chain round its neck is pulled.

The manufacture of robots is rapidly increasing in many countries in the world today, taking over more and more work formerly done by human beings. Can Sri Lanka cope with robots? My answer is Yes. Though we do not have large industries, there is much work that could be done by robots. Thinking alone is not going to help us. We should aim to create useful robots in Sri Lanka in the near future.

No scientist thinks in formulae. - Einstein

STATISTICAL ASTROLOGY

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It is seen that the number of non scientists who believe in astrology is declining; while some scientists, who were sceptical about such pseudo sciences a few decades ago, are beginning to show an interest in this subject.

Do the astrologers make their predictions according to a 'scientific' basis ?

An astrologer who hopes to predict the future of a particular person, first calculates the position of the planets as seen at the time and place of birth of that person. Then his future is predicted based on these planetary positions.

Astrologers in the distant past may have made careful studies of the correlations (which we, for the purpose of this article, assume to exist) between the characteristics of certain people, the incidents that took place in their lives, and the apparent positions of the planets at their respective moment of birth. In doing these, the astrologers may have identified some similarities and patterns. They may have also drawn up charts to illustrate these correlations. After generations of such studies, one would imagine that astrologers became quite proficient in forecasting the future of people, although it is hard to imagine that such predictions could be perfectly accurate.

Modern day statisticians follow a similar procedure in making predictions. Suppose a statistician has to determine the average height of an adult Sri Lankan. He will first take a random sample of the adult population of this country and calculate the average height of a person in the sample. He will then be able to ascertain the required statistic at some α% level of significance. (The smaller the α , the better the estimate or prediction.) α depends on the method of calculation. This branch of statistics is termed Statistical Inference.

If astrological predictions could be made in this manner with a certain level of significance attributed to them, it would be more 'scientific'.

Will this be called Statistical Astrology?

Two things fill the mind with ever new and increasing wonder and awe - the starry heavens above me and the moral law within me.

- Kant

විශ්වය හා මම

උපාලි ජයසිංහ
2 වැනි වසර

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THE GENIUS OF SIR ISAAC NEWTON

Deepthi Amarasuriya
3rd year (Mathematics Sp.)

A student of physics and mathematics hardly fails to be amazed at the regularity with which one comes across the name Sir Isaac Newton. The discoveries of this extraordinarily brilliant scientist opened many areas of study ranging from optics and celestial mechanics to integral and differential calculus. Indeed, Newton is ranked along with Archemedes and Einstein as one of the three greatest mathematicians of all time.

Yet, he was a humble man. Newton realised that his findings did not encompass all of nature, that he had not by any means uncovered all the laws by which the universe is governed. He himself said, "I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the sea shore, and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary whilst the great ocean of truth lay all undiscovered before me."

Isaac Newton was born in the English village of Woolsthorpe to Mrs. Hannah Newton on Christmas day, 1642. Mr. Isaac Newton, senior, had died two months before the birth of his son. After Mrs. Newton's second marriage in 1645 the boy was taken care of by his maternal grandmother who kept him at her house and sent him to a grammar school in the nearby village of Grantham. Isaac did not take much interest in his school books and was not very strong physically but after unexpectedly emerging the victor from a tussle with the school bully the young boy gained confidence in himself and improved amazingly in his studies.

When the boy was 14, his step-father died and Mrs. Hannah Newton returned to Woolsthorpe. Her son was taken out of school to assist her on her farm, but his interest in mathematics far outweighed his enthusiasm for farm work. It is said that his mother entrusted him with her herd of cattle; but no sooner than Newton took the cattle to the pasture than he seated himself under a tree, note book and pen in hand and became lost in his thoughts while the cattle strayed far and wide.

His uncle William Ayscough a member of Trinity College, Cambridge happened to pay a visit to Mrs. Newton and realizing the extraordinary potential of her son persuaded her to send him back to school to prepare for the Cambridge entrance.

Newton entered Trinity College, Cambridge in 1661. Nothing much is known of his days as an undergraduate except that he was elected as a scholar in 1664 and graduated later on that year.

In 1665, disaster struck. The dreaded Bubonic plague spread throughout London, killing thousands of people and causing the panic stricken survivors to flee the city. As the epidemic spread, Newton left Cambridge for his native Woolsthorpe, and in the peace and quiet of his mother's farm he turned his prolific mind to the problems of physics and mathematics.

The years 1665-1667 were ones of brilliant creativity. Within the short span of two years the young Newton worked out the binomial theorem, discovered the elements of integral and differential calculus, investigated the phenomenon of gravitation and commenced experiments on optics.

"In the year 1666... I procured me a triangular glass prism to try therewith the celebrated phenomena of colours." Newton wrote, "And in order thereto, having darkened my window-shuts to let in a convenient quantity of the sun's light, I placed my prism at its entrance, that it might be thereby refracted to the opposite wall.

It was at first a very pleasing [diversion] to view the vivid and intense colours produced thereby;...." recorded Newton, but not being content merely with appraising the beauty of the visible spectrum he began to study it in detail. He showed conclusively that white light is made up of seven colours and realized that the colour of an object is determined by the light it reflects. He studied diffraction and investigated the phenomenon of chromatic aberration. Small wonder that he is considered as one of the founders of modern optics.

The most popular anecdote on Newton concerns the incident which led him to discover the law of universal gravitation. The apparently commonplace observation that apples always fell onto the ground made him ponder on the force of attraction between masses. He showed mathematically that the moon was kept in orbit around the earth because its inward fall was exactly counter-balanced by its centrifugal force and he formulated the inverse square law of gravitation.

Newton was undoubtedly one of the greatest of scientists but not all the preliminary insights required for obtaining his results can be attributed solely to him. Newton gained considerably from the inspirations of others but where lesser men had groped in the semi-darkness of speculations and half truths, his clear mind extracted the finer points of their arguments and with conclusive work of his own, established them as laws of science. The ancient Greeks for instance, realized that a curve could be considered as a sequence of infinite points and that it was possible to think of an area as a collection of infinitely thin slices. Other mathematicians of Newton's time also made use of the notion of infinitesimals in their calculations. But it was left to Newton to develop and present these ideas as the calculus.

When he showed his professor Isaac Barrow some of his results, Barrow was so impressed by Newton's work that on retiring in 1669 he got Newton, then only 27, appointed to his chair at Cambridge as the Lucasian Professor of Mathematics.

His duties as a professor were light. He could freely select the topic of the weekly lecture he was required to give from several fields including geometry, optics, mechanics, calculus and astronomy. This together with a comfortable income enabled him to pursue his interests.

The invention of the world's first reflecting telescope caused him to be elected in 1672 as a fellow of the Royal Society of London, an elite group of learned men of his day. The theory of light and colour which he presented before this Society brought both praise and criticism from his peers. The latter disturbed the sensitive Newton to a great extent, especially as the well known physicists Robert Hooke and Christian Huygens did not agree with some of his conclusions. Also, Hooke accused Newton of borrowing his own ideas on the corpuscular theory of light. In a letter to Henry Oldenburg, the Secretary of the Society, Newton wrote ".... I see a man must either resolve to put out nothing new or to become a slave to defend it."

Whatever happiness and exuberance Newton experienced on being elected as a member of the Royal Society was now gone. Newton, always suspicious and reticent, drew even more into his shell. Immersed in scientific thoughts he often forgot to eat and hardly slept, remaining in his room for several days at a time. He seems to have had a nervous breakdown from which he appears to have recovered quickly. In this solitary confinement he produced his masterpiece - The Principia.

The Philosophae Naturalis Principia Mathematica to give its complete title, is a monumental work in three books. Book I deals with motion under the idealized absence of resistance. Newton's famous laws of motion make their first appearance in book I and are later expanded in book III. Although credit goes to Galileo for the idea that the acceleration of a body is a measure of the forces acting upon it, Newton went one step further by introducing the concept of mass. He showed that the mass of a body is different from, but is proportional to its weight and he clearly recognized mass as a property of all bodies, a property dynamically fundamental and measurable. With his laws of motion, Newton defined the concepts of mass, inertia and force, and their relation to acceleration and velocity - thus giving rise to the branch of physics known as dynamics.

The second volume of Principia is mainly concerned with the motion of fluids, taking resistance into consideration. In this book he showed that Descartes' hypothesis of vortices is, in addition to being non essential to a scientific analysis of planetary motion, is incapable of being subject to mathematical demonstration.

The law of science which is almost synonymous with the name Sir Isaac Newton - the law of universal gravitation, is announced in book III. This law is the statement that every particle of matter attracts every other mass in the universe with a force that is directly proportional to the product of their respective masses, and inversely proportional to the square of the distance between them. Armed with this law and the powerful tool of calculus, Newton set out to solve many problems that had confounded astronomers.

He computed the earth's density to be between five and six times that of water (the modern estimation gives this figure as being 5.52 times the density of water). He investigated the phenomenon of the earth's tides, showing them to be the result of the gravitational pull of the sun and the moon on the earth. In calculating the orbits of the planets round the sun, he pioneered the study of planetary perturbation which is concerned with the deviation of a planet in its orbit round the sun due to the gravitational effect of the third body. It is of interest to know that the advances later made in this field of study led to the discovery of the planet Neptune, in 1846. Newton deduced more or less empirically that the earth is an oblate spheroid, and he also demonstrated that a sphere attracts other bodies as if all its mass were concentrated at its centre. He calculated the precession of the equinoxes and the orbit of comets. Above all, Newton was responsible for originating and perpetuating the deterministic world view that dominated Western scientific thinking until the advent of quantum mechanics in the twentieth century.

Newton completed the writing of The Principia in an incredibly short interval of 18 months. During this period, he toiled day and night. He hardly left the confines of the university, and while there spent almost all his time at his work. A distant relative of Newton's who served as his secretary and manuscript copyist recorded that "....Oftentimes he forgot to eat at all, so that going into his chambers I have found his [meals] untouched, of which when I have reminded him he would reply 'Have I ? ' and then making to the table would eat a bit or two standing, for I cannot say I ever saw him sit at the table by himself." He also noted that the woman who delivered Newton's meals "....sometimes found both dinner and supper scarcely tasted of.." and "...has very pleasantly gone away with [them] ."

The proofs in The Principia are given in the form of equations in geometry although they could have been obtained by means of the calculus. It is believed that Newton had in fact utilised the calculus in arriving at his proofs but reworked his results using geometry. Newton intentionally made The Principia abstruse in order (as he wrote to a friend) "...to avoid being bated by little smatterers in mathematics ....but yet so as to be understood by able mathematicians."

Newton never quite recovered from the physical and mental strain he underwent during the writing of The Principia. The almost superhuman labours that Newton put into the writing of The Principia seem to have diminished his creative powers to a considerable extent.

During this period his interests took a new turn. The absent minded scientist in the proverbial ivory tower, who a few years back had scrupulously avoided human contact now becme involved in political and religous matters. He became a chronologist of Bibical history and was appointed as the Master of the Mint.

But it must not be thought that he abandoned science completely. Newton held the Presidency of the Royal Society of London for the 25 year period from 1703 to 1727 and personally supervised the publication of the

Greenwich Observations written by the first Royal Astronomer - John Flamsteed. It is said of Newton that on returning home from the Mint, he worked out problems posed by rival mathematicians - each time in the course of a single evening.

Early in 1727 Newton fell ill. He died later that year, at the age of 85 and was buried in Westminister Abbey. On his tomb is inscribed perhaps the greatest tribute that was paid to this Genius :

" Nature and nature's laws lay hid in night,
God said ' Let Newton be ' and all was light. "

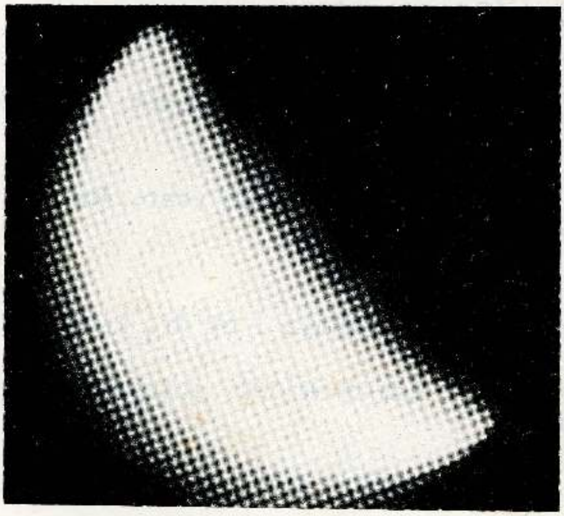
Newton's philosophy was based solely on a mechanical world view and embodied the concept of determinism. He believed that all natural phenomena could be explained in terms of forces which acted on particles comprising tangible bodies. Newton propounded that once the configuration of a system of particles and all the forces acting on it were completely known (for he accepted that such physical measurements were in principle, possible), the behaviour of the bodies under consideration could be determined with certainty.

Before the 20th century, physicists busied themselves exclusively with expanding the range of applications of Newtonian mechanics, while only a handful of philosophers of this period questioned his world view. The 18th century mathematicians Hamilton and Lagrange recast Newton's mechanics in an analytical mould, leaving its fundamental concepts intact. Even the great Einstein dismissed the non-deterministic approach to mechanics with his famous remark that God does not play dice with the universe.

Not all experimental results could be interpreted in terms of the so called classical mechanics of Newton. A completely different formulation of how matter and waves behave was required to explain the anomalies between the outcomes of particular experiments and what the classical theory predicted. It required the brilliant insight of Planck, Bohr, Heisenberg, Dirac, Schrodinger and others to put forward a radically new theory based on a probabilistic world view.

The postulates of quantum mechanics are strange, to say the least. They are formulated on abstract mathematical principles, not on empirical observations. Quantum theory attempts to describe how the microscopic subatomic world behaves. Its results hold in the macroscopic domain too, although departures from Newtonian mechanics becomes negligible when applied to objects and motions of such dimensions as are 'directly' perceived with the five senses of a human being. Hence for our day-to-day work Newtonian mechanics is good enough. Many of Newton's theories have now been improved if not discarded outright. But it goes without saying that Sir Isaac Newton strived as few mortals have done to solve the eternal quest of man - to unravel the secret of the universe - and was able to see much of it, where all the others before him failed.

TOTAL LUNAR ECLIPSE - 4TH MAY 1985



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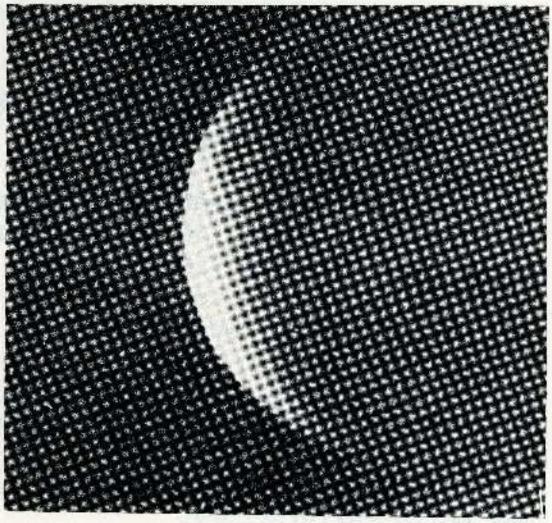

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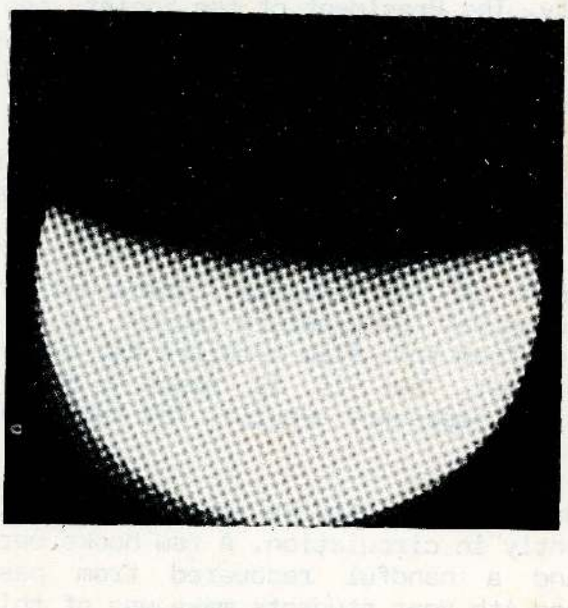

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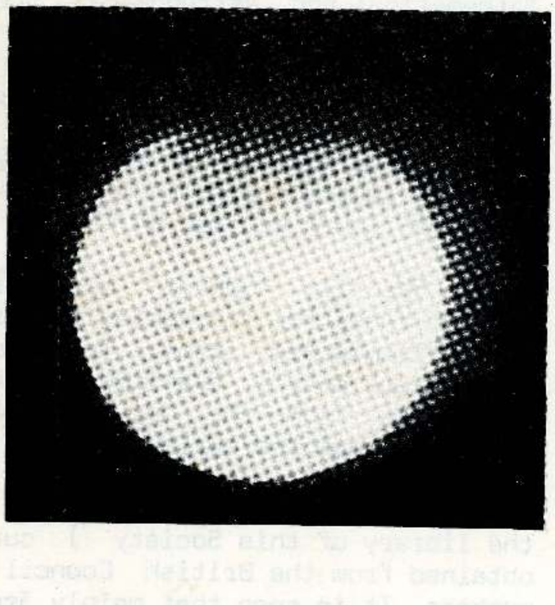

The above photographs were taken using a photographic camera in conjunction with the $4\frac{1}{4}$ inch refractor at the Observatory, University of Colombo.

MATHEMATICAL AND ASTRONOMICAL SOCIETY
ANNUAL REPORT 1984 - 1985

Deepthi Amarasuriya (Secretary)

The academic year 1984/85 saw a marked improvement in the activities of the Society.

4 general meetings were held in the course of this year. The main items on the respective agendas were as follows.

1. A video film on fibre optics.
2. A video film on computers.
3. The Universe we Inhabit - A lecture (in Sinhala) by Dr. L.N.K. de Silva.
4. Screening of slides on objects of Astronomical interest with explanations given by the Secretary.

8 committee meetings were also held. The attendence at all meetings was quite satisfactory.

The Freshers' Welcome was held at the Physics Lecture Theatre on the 27th of March 1985. Prof. P.W. Epasinghe and Prof. O.W. Jayaratne each spoke briefly on this occasion, highlighting the importance of the Mathematical and Astronomical Society. The President of the Society too, addressed the large gathering of freshers who were present. A film on space travel was screened (with the courtesy of the Russian Cultural House), and refreshments were provided.

A few observation evenings were held when the weather permitted. 10 students stayed overnight at the Astronomical Observatory to photograph the total lunar eclipse on May 4th 1985.

The film 'Sphinx' was screened at the Liberty Cinema Hall to collect funds for renovating the Astronomical Observatory which is in a very dilapidated state. The proceeds from this film totalled Rs. 4,889. This covers only a fraction of the cost, but it is hoped to obtain assistance from the University for the necessary repairs. The telescope tubes were painted early in 1985.

The librarian has reported that there are 114 books (belonging to the library of this Society) currently in circulation. A few books were obtained from the British Council and a handful recovered from past members. It is seen that mainly 3rd and 4th year students make use of this facility.

A novel feature introduced by Prof. P.W. Epasinghe has been the monthly 'Problem Corner'. This is a refreshing approach to mathematical problem solving and a welcome way of testing the students' abilities.

We convey our sincere thanks to Prof. P.W. Epasinghe, the Patron of our Society for his interest in our activities and his valuable advice, and Prof. V.K. Samaranayake for his permitting us the use of the word processor at the Computer Centre in printing this magazine. We are deeply grateful to Dr. Mrs. L.H. Liyanage for serving as the Senior Treasurer in the year under consideration and to Dr. L.N.K. de Silva for his thought provoking lecture. We also take this opportunity to thank the clerical staff of the Dept. of Mathematics for their ungrudging help.

Last, but not least a thank you to all members of this Society who helped in making this year a success.

The essential thing in science is for the scientist to think up a theory. There is no way of mechanizing this process ; there is no way of breaking it down into science factory. It always requires human imagination and indeed in science we pay the highest respect to creativity, to originality ...... We do not honour scientists for being right ; it is never given to anybody to be always right. We honour scientists for being original, for being stimulating, for having started a whole line of work.

- Sir Herman Bondi

(An originator of the Steady State theory of the universe)

MATHEMATICAL & ASTRONOMICAL SOCIETY
UNIVERSITY OF COLOMBO

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Editorial Sub-Committee:

Since the work on this magazine could not be completed by the end of the 1984-85 academic year, the Council of the 1985-86 academic year appointed the following Editorial Sub-Committee to do the remaining work.

1 Miss. D. Amarasuriya
2 Mr. H.J.S. Fernando
3 Mr. L.H.S. Peiris
4 Mr. A. Fonseka
5 Mr. D.L.L.W. Peiris

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* Prof. P.W. Epasinghe for his help and encouragement.

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* The advertisers for their kind patronage.

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