

MATHEMATICS

PART - I



Grade 9



Educational Publications Department

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MATHEMATICS

Part - I

Grade

9

Educational Publications Department

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The National Anthem of Sri Lanka

Sri Lanka Matha

Apa Sri Lanka Namō Namō Namō Namō Matha

Sundara siri barinee surendi athi sobamana Lanka

Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya

Apahata sepa siri setha sadana jeewanaye matha

Piliganu mena apa bhakthi pooja Namō Namō Matha

Apa Sri Lanka Namō Namō Namō Namō Matha

Oba we apa vidya

Obamaya apa sathya

Oba we apa shakthi

Apa hada thula bhakthi

Oba apa aloke

Apaga anuprane

Oba apa jeevana we

Apa mukthiya oba we

Nava jeevana demine, nithina apa pubudukaran matha

Gnana veerya vadawamina regena yanu mena jaya bhoomi kara

Eka mavakage daru kela bevina

Yamu yamu vee nopama

Prema vada sema bheda durerada

Namō, Namō Matha

Apa Sri Lanka Namō Namō Namō Namō Matha

ஈபி வெலு லக மலககெ டுருவெர்
லக நிலசெனி வெசெனா
லக பாலுநி லக ருடெரசு வெ
ஈப கக குல டுலனா

லகலெநி ஈபி வெலு சேயுசுரு சேயுசுரெசெர்
லக லெச லநி லுலெனா
சீலந் லன ஈப மெம நிலசெ
சேயுடென சிடுச யுலு வெ

சுலமல ம மெந் கருனா குனெநி
வெலெ சமடு டுலெநி
ரந் மீனெ லுலு நேய ல லச ம ய சுபனா
கிசி கல நேயம டுலனா

- ஈயநந் டு சமரகெந் -

ஓரு தாய் மக்கள் நாமாவேம்
ஓன்றே நாம் வாமும் இல்லம்
நன்றே ஁டலில் ஓடும்
ஓன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவேம்
ஓன்றாய் வாமும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

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யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Foreword

The mission of education is to provide for the future and to ensure that the younger generation can grow up as citizens able to face the challenges of the future with confidence. The changes instituted in the grade nine school curriculum with these objectives in mind came into effect in 2010. This textbook is a reprint of it.

As the government has to disburse an unbearable amount of its revenue on free textbooks, it is your duty to make the maximum use of this textbook and protect it well so that it can be reused. Then you can be proud that you have added your mite to the national assets.

I wish to take this opportunity to thank the writers, the editors, the members of the evaluation boards, the officers of the Educational Publications Department and all the others who have contributed to the compilation of this book.

W.M.N.J. Pushpakumara
Commissioner General of Educational Publications

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Rounding Off and Scientific Notation

By studying this chapter you will be able to achieve the following competencies.

- ★ Comprehending easy methods of writing and reading numbers.
- ★ Writing in scientific notation, a number given in general form.
- ★ Expressing in general form, numbers given in scientific notation.
- ★ Rounding off a number to a given power of ten.
- ★ Rounding off decimals.

1.1 Scientific Notation

A picture of a scene from Sinharaja forest which is a World Heritage is seen here.

The area of 'Sinharaja' is expressed there as 9.3×10^3 ha.

The average land area of Sinharaja forest is 9 300 hectares. We know that the number 9 300 can be written as

$$\begin{aligned} & 930 \times 10 \\ \text{or} & 93 \times 100 \\ \text{or} & 9.3 \times 1000 \end{aligned}$$



Sinharaja Forest Area 9.3×10^3 ha

9.3×1000 is 9.3×10^3 . When this land area is written in the form 9.3×10^3 it is very easy to indicate this number, as shown below.

$$\begin{array}{ccc} 9.3 & \times & 10^3 \\ \downarrow & & \downarrow \\ \text{Number between} & & \text{The power of 10} \\ \text{one and ten} & & \end{array}$$

Expressing a number as a product of 1 or a number between 1 and 10 and a power of 10, is known as denoting a number in scientific notation.

This notation can be generally expressed as $P = a \times 10^n$, $n \in \mathbb{Z}$

$$\text{Here } 1 \leq a < 10$$



Mass of the earth is approximately 6.0×10^{24} kg

Accordingly, when the number 9 300 is indicated in scientific notation it is written as 9.3×10^3

Example 1

Write 725 000 in scientific notation

$$\begin{aligned} 725\ 000 &= 7.25 \times 100\ 000 \\ &= \underline{\underline{7.25 \times 10^5}} \end{aligned}$$

Example 2

Write 25.3 in scientific notation

$$\begin{aligned} 25.3 &= 2.53 \times 10 \\ &= \underline{\underline{2.53 \times 10^1}} \end{aligned}$$



Exercise 1.1



- (1) Complete the table given below expressing in scientific notation the numbers written in general form in the first column.

	Number in general form	As a product of 1 or a number between 1 and 10, and a power of ten	Number in scientific notation
(i)	9 300	9.3×1000	$9.3 \times \text{---}$
(ii)	500	5.0×100	$5.0 \times \text{---}$
(iii)	32 000	-----	-----
(iv)	30 500	$3.05 \times 10\ 000$	-----
(v)	7 250	-----	-----
(vi)	1 000 000	-----	-----

- (2) Write the numbers given below in scientific notation.

(i) 6 000 (ii) 72 000 (iii) 12 500 (iv) 33 300 (v) 275 000
 (vi) 186 000 (vii) 10 000 (viii) 21 (ix) 111 (x) 3 333

- (3) Write the number six hundred thousand in (i) general form (ii) scientific notation
- (4) The land area of Sri Lanka is about 65 610 square kilometres. Write this number in scientific notation.
- (5) If 12 kg of rice is needed for a pupil for monthly consumption, find the quantity of rice needed for a hostel with 200 pupils, for one month. Write this number in scientific notation.
- (6) Sri Lanka produces 810 000 kg of tea per day for export. Write this number in scientific notation.

Can you express the number 1.235 in scientific notation?

1.2 Writing a number less than 1 in scientific notation

Observe how the following numbers are written as powers of ten

$$\begin{aligned} 1000 &= 10^3 \\ 100 &= 10^2 \\ 10 &= 10^1 \\ 1 &= 10^0 \\ 0.1 &= \frac{1}{10} = 10^{-1} \\ 0.01 &= \frac{1}{100} = 10^{-2} \end{aligned}$$

Accordingly, decimal numbers have negative indices. Also according to the pattern of powers of ten
 $0.1 = 10^{-1}$, $0.01 = 10^{-2}$

Example 3

Express 0.5 in scientific notation

$$0.5 = \frac{5}{10} = 5 \times \frac{1}{10} = \underline{\underline{5.0 \times 10^{-1}}}$$

Example 4

Express 0.72 in scientific notation

$$0.72 = \frac{72}{100} = \frac{7.2}{10} = 7.2 \times \frac{1}{10} = \underline{\underline{7.2 \times 10^{-1}}}$$

Example 5

Write 0.05 in scientific notation

$$0.05 = \frac{5}{100} = 5.0 \times \frac{1}{100} = \underline{\underline{5.0 \times 10^{-2}}}$$

Exercise 1.2

1. Denote the decimal numbers in the first column in scientific notation and complete the table given below.

Decimal number	As a product of 1 or a number between 1 and 10; and a number which is a power of 10	Decimal number in scientific notation
(i) 0.3	$\frac{3}{10} = 3 \times \frac{1}{10^1}$	$3.0 \times \text{-----}$
(ii) 0.7	-----	-----
(iii) 0.27	$\frac{27}{100} = 2.7 \times \frac{1}{10^1}$	-----
(iv) 0.35	-----	-----
(v) 0.02	$\frac{2}{100} = 2.0 \times \frac{1}{10^2}$	-----
(vi) 0.04	-----	-----

2. Write in scientific notation

(i) 0.5

(ii) 0.25

(iii) 0.05

(iv) 0.032

(v) 0.00021

1.3 Writing in general form numbers given in scientific notation

When converting a number written in scientific notation to a number in general form, the number 1 or the number between 1 and 10 is multiplied by the corresponding power of 10.

Example 6

Write in general form 1.2×10^3

$$\begin{aligned} &1.2 \times 10^3 \\ &= 1.2 \times 1000 \\ &= 1200.0 \\ &= \underline{\underline{1200}} \end{aligned}$$

Example 7

Write 3.05×10^5 in general form

$$\begin{aligned} &3.05 \times 10^5 \\ &= 3.05 \times 100\,000 \\ &= 305\,000.00 \\ &= \underline{\underline{305\,000}} \end{aligned}$$

Example 8

Write 2.0×10^{-2} in general form

$$\begin{aligned} &2.0 \times 10^{-2} \\ &= 2.0 \times \frac{1}{100} = \frac{2}{100} \\ &= \underline{\underline{0.02}} \end{aligned}$$

A light year is the distance light travels in one year.

$$\begin{aligned} \text{A light year} &= 9.5 \times 10^{12} \text{ km} \\ &= 9500\,000\,000\,000 \end{aligned}$$

Exercise 1.3

1. Write in general form

(i) 2.0×10^2

(ii) 7.0×10^4

(iii) 5.2×10^3

(iv) 7.5×10^4

(v) 8.3×10^5

(vi) 7.25×10^3

2. The diameter of Mercury, the planet which is closest to the sun is 5×10^3 km. Write this in general form.

3. Write 5.2×10^{-1} in general form.

4. Find the larger number out of the two numbers 7.25×10^3 and 2.7×10^4 . Give reasons for your answer.

Activity 1



Refer some books and collect information connected with numbers written in scientific notation. Select 5 of them and write them in general form.

1.4 Rounding off numbers

A part of a dialogue between a mother and a child who was preparing to go on an educational tour from Kurunegala to Colombo, is given below.

Mother, what is the distance we have to cover, up and down during our educational tour? How much will be the expenses?



The distance up and down of the tour will be almost 200 kilometres. So the expenses will also be about Rs. 300



Here the numerical values 200 kilometres and Rs 300 are approximate values and not exact values. Since the distance from Kurunegala to Colombo up and down is $93 \text{ km} + 93 \text{ km} = 186 \text{ km}$ the distance she expressed is justifiable. The expenses Rs 300 may also be justifiable.

The numerical values that we express during conversations in day to day life are mostly such approximate values.

When selecting an approximate value for a number, communication will be easy if such a value is a multiple of 10.

Expressing a number as an approximate value according to a rule is called rounding off.

1.5 Rounding off numbers to the nearest 10

Examine the table given below which shows the rounding off of a few numbers. Here attention is paid to the digit in the units place of the number, whether it is 5, greater than 5 or less than 5.

Number	Multiple of 10 less than the number	Multiple of 10 greater than the number	Value after rounding off	Reason for rounding off
24	20	30	20	Rounding off to the multiple of 10 less than the number, as the digit in the units place of 24 is 4 which is less than 5.
76	70	80	80	Rounding off to the multiple of 10 greater than the number, as the digit in the units place of 76 is 6 which is greater than 5.
195	190	200	200	Rounding off to the multiple of 10 greater than the number as the digit in the units place is 5.
3152	3150	3160	3150	Rounding off to the multiple of 10 less than the number as the digit in the units place is 2 which and is less than 5.

Accordingly, rounding off will be done as follows;

$24 \longrightarrow 20$ $76 \longrightarrow 80$
 $195 \longrightarrow 200$ $3152 \longrightarrow 3150$

Steps of rounding off a number to the nearest 10

- (ii) Examining whether the digit in the units place is 5 less than or greater than 5.
- (i) Identifying the nearest multiple of 10 less than or greater than the relevant number.
- (iii) Rounding off the number to the multiple of 10 greater than the number if the digit in the units place is 5 or greater than 5; to the multiple of 10 less than the number if the digit in the units place is less than 5.

Exercise 1.4

(1) Round off to the nearest 10

(i) 28 (ii) 73 (iii) 61 (iv) 99 (v) 8

(2) Round off to the nearest 10

(i) 127 (ii) 355 (iii) 805 (iv) 4 003 (v) 5 008

- (3) The height of Adam's Peak is 2243 m. Round off this to the nearest 10 metres
- (4) The length of Malwathu Oya is 164 km. Round off this to the nearest 10 Kilometres
- (5) The amount of money spent for buying vegetables from a market is Rs 347. Round off this expenditure to the nearest Rs 10.

1.6 Rounding off numbers to the nearest 100

When rounding off a number to the nearest 100, the digit in the tens place is considered, whether it is 5, greater than 5 or less than 5. Examine the table given below.

Number	Multiple of 100 less than the number	Multiple of 100 greater than the number	Reason for rounding off	Value after rounding off
182	100	200	Rounding off to the multiple of 100 greater than the number as the digit in the tens place is 8	200
552	500	600	Rounding off to the multiple of 100 greater than the number as the digit in the tens place is 5	600
1239	1200	1300	Rounding off to the multiple of 100 less than the number as the digit in the tens place is 3	1200

Accordingly, rounding off will be done as follows.

182 \longrightarrow 200 , 552 \longrightarrow 600 , 1239 \longrightarrow 1200

1.7 Rounding off numbers to the nearest 1000

When rounding off a number to the nearest 1000 we examine whether the digit in the hundred's place is 5, greater than 5 or less than 5.

Example 9

Number		Value after rounding off	Reason
(i)	2439	2000	As the digit in the hundred's place is 4 it is rounded off to the lower multiple of 1000
(ii)	7621	8000	As the digit in the hundred's place is 6 it is rounded off to the upper multiple of 1000
(iii)	12300	12000	As the digit in the hundred's place is 3 it is rounded off to the lower multiple of 1000

Example 10

Round off 7358

- (i) to the nearest 10
- (ii) to the nearest 100
- (iii) to the nearest 1000

7358 \longrightarrow 7360 (to the nearest 10)
7358 \longrightarrow 7400 (to the nearest 100)
7358 \longrightarrow 7000 (to the nearest 1000)

Activity 2

If a number when rounded off to the nearest 10, 100, and 1000 is 10 000, write the set of numbers to which it belongs.

Exercise 1.5

- (1) Round off the following numbers to the nearest 100
 - (i) 97
 - (ii) 132
 - (iii) 1 750
 - (iv) 5 280
 - (v) 2 999
- (2) Round off to the nearest 1000
 - (i) 1 999
 - (ii) 5 280
 - (iii) 7 199
 - (iv) 6 666
 - (v) 15 520
- (3) Round off 1 827
 - (i) to the nearest 10
 - (ii) to the nearest 100
- (4) Round off 37 295
 - (i) to the nearest 10
 - (ii) to the nearest 100
 - (iii) to the nearest 1 000
- (5) The distance from Colombo to Jaffna is 396 km. Round off this to the nearest 100 kilometres.
- (6) The distance from Anuradapura to Yala is 401 km. Round off this to the nearest 100 kilometres.
- (7) The land area of Udawalawa sanctuary is 30 821 ha. Round off this to the nearest 1000 ha.

1.8 Rounding off decimal numbers to the nearest whole number

Let us round off the numbers 2.3, 2.5, 2.8 to the nearest whole number.
Examine the position of these numbers on a number line.



Number	Position	The nearest whole number to which it is rounded off
2.3	The nearest whole number is 2	2
2.5	The position is half way between 2 and 3 (Will be rounded off to the upper whole number because of 5)	3
2.8	The nearest whole number is 3	3

1.9 Rounding off a decimal number to a directed decimal place

Example 11

Round off 5.37 to one decimal place

$$\begin{array}{r} 5.37 \\ \hline 5.4 \end{array} \quad \left[\begin{array}{l} \text{As the digit in the second decimal place is 7} \\ \text{which is greater than 5, 1 is added to the first} \\ \text{decimal place} \end{array} \right]$$

Example 12

Round off 4.351 to 2 decimal places.

$$\begin{array}{r} 4.351 \\ \hline 4.35 \end{array} \quad \left[\begin{array}{l} \text{As the digit in the third decimal place is less} \\ \text{than 5, it is neglected. 1 is not added} \end{array} \right]$$

Example 13

Round off 2.537

- (i) to one decimal place
- (ii) to two decimal places
- (iii) to the nearest whole number

(i) $\begin{array}{r} 2.537 \\ \hline 2.5 \end{array}$ (to one decimal place)

(ii) 2.537
 2.54 (to 2 decimal places)

(iii) 2.537
 3 (to the nearest whole number)

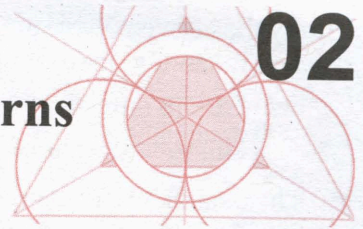
Exercise 1.6

- (1) Round off 3.76
 - (i) to the nearest whole number
 - (ii) to one decimal place
- (2) Perera's weight is 62.8 kg. Round off this weight to the nearest kilogram.
- (3) Round off Rs 7.85 to the nearest Rupee.
- (4) A mile is equal to 1.609 kilometres. Round off this to one decimal place.
- (5) If it is given that $\pi = 3.14159$, round off the value of π
 - (i) to 2 decimal places
 - (ii) to 3 decimal places
- (6) Usain Bolt of Jamaica has completed the 100 m flat race event of Olympic Games, which is considered as the fastest race in the world in 9.64 seconds. Round off this time to one decimal place.
- (7) The longest tunnel in the world which is 53.85km long is situated between the islands, Honstu and Hokindo. Write giving reasons methods of expressing the length of the tunnel to a friend.



Number Patterns

02



By studying this chapter you will be able to achieve the following competencies.

- ★ Identifying how number patterns are formed and finding the general term of the patterns.
- ★ Finding the value of any term when the general term of a number pattern is given and doing calculations associated with it.
- ★ Making calculations easy in different practical situations by using number patterns.

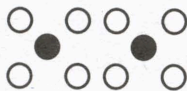
2.1 Identifying number patterns

We can form various number patterns by assigning numerical values to groups of material arranged in some order.

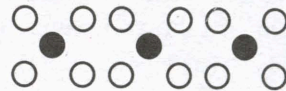
Given below is a pattern prepared using buttons of colours black and white coloured buttons.



(i)



(ii)



(iii)

Pattern number	i	ii	iii	iv	v	vi
Number of black buttons	1	2	3	4	5	6
Number of white buttons	4	8	12	16	20	24

The rest of the pattern can be developed by observing the numbers already given.

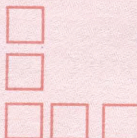
By doing the following exercises you will be able to improve your knowledge of number patterns

Exercise 2.1

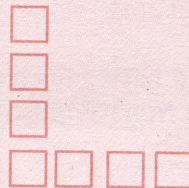
(1)



(i)



(ii)

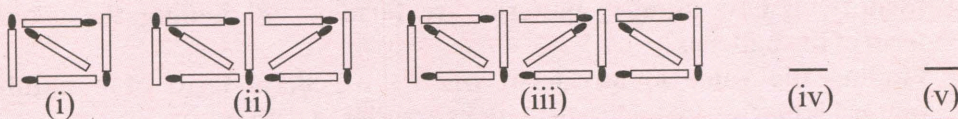


(iii)

Study the above pattern and write down the number of small squares needed for stages (iv) and (v) in the table given below.

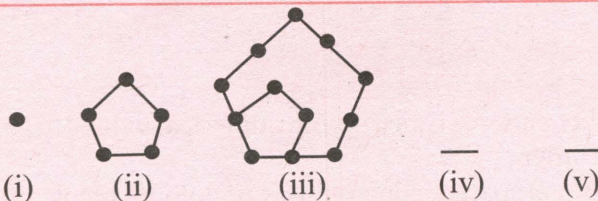
Pattern number	i	ii	iii	iv	v
Number of squares	3	5	7	—	—

(2) The diagram given below shows the first three stages of a pattern formed with matchsticks of equal length. Accordingly write the number of matchsticks needed for the next two stages.



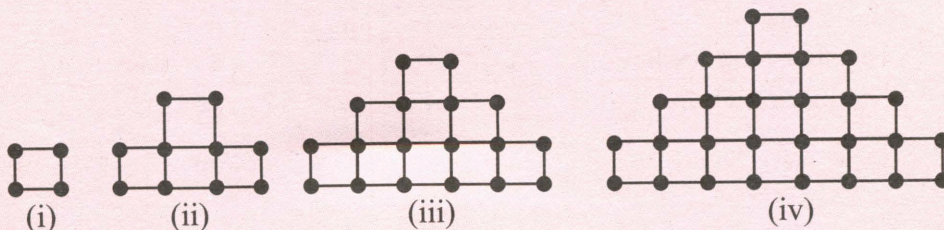
Pattern number	i	ii	iii	iv	v
Number of matchsticks needed	5	9	13	—	—

(3)



The first three stages of a sequence of a pattern made using dark dots in the shape of a pentagon are shown above. Draw the necessary shapes with the dark dots suitable for the next two stages and write down the number of dots needed.

(4) Shown below is a pattern made by joining dots in the shape of squares.



Study the above pattern and fill in the table given below.

Pattern number	i	ii	iii	iv	v	vi
No. of rows of squares	1	2	3	4	—	—
Total number of squares	1	4	9	16	—	—
Total number of dots	4	10	18	28	—	—

By now you must have gained an understanding about number patterns using various diagrams. You have also learnt in previous grades about number patterns of the forms given below.

- Even numbers
- Odd numbers

2, 4, 6, 8, 10,

1, 3, 5, 7, 9,

- Multiples of three 3, 6, 9, 12, 15,
- Multiples of four 4, 8, 12, 16, 20,
- Square numbers 1, 4, 9, 16, 25,
- Triangular numbers 1, 3, 6, 10, 15,

2.2 Terms of a number pattern

Let us further investigate about number patterns.

- ◆ The term written first in a number pattern is called the **first term**.
Accordingly, the first term of the pattern 3, 5, 7, 9, is 3.
- ◆ In a number pattern, the terms that are placed next to each other are called **successive terms**.

(Here terms 5 and 7, 7 and 9, 9 and 11 are successive terms.)

3, 5, 7, 9, 11, 13,

The difference between two successive terms of a number pattern can be obtained as follows.

$$\text{The difference between two successive terms} = (\text{the value of the consequent term}) - (\text{the value of the antecedent term})$$

Example 1

Antecedent term Consequent term

(i) 3, 5, 7, 9, 11, 13, 15, 17,

The difference between the successive terms 13 and 15 = $15 - 13 = 2$

(ii) 100, 90, 80, 70,

The difference between the successive terms 90 and 80 = $80 - 90 = -10$

Exercise 2.2

Complete the table given below.

Number Pattern	First Term	The difference between the successive terms	The difference between the successive terms is equal /unequal
(1) 4, 6, 8, 10, ----			
(2) 30, 40, 60, 70, ----			
(3) $\frac{1}{2}$, $1, 1\frac{1}{2}$, $2, 2\frac{1}{2}$, ----			
(4) -12, -10, -8, -6, ----			
(5) 0.3, 0.8, 1.3, 1.8, ----			
(6) 20, 23, 28, 31, ----			

2.3 Obtaining the general term of a number pattern

Example 2

By examining the table given below, we can comprehend how the number pattern 3, 5, 7, 9, 11 is formed.

3, 5, 7, 9, 11 (The difference between the terms is 2)

$\underbrace{\quad\quad}_{+2}$ $\underbrace{\quad\quad}_{+2}$ $\underbrace{\quad\quad}_{+2}$ $\underbrace{\quad\quad}_{+2}$

Serial number	Value of the term	Value of the term in terms of the first term
1	3	$3 = 3 + 2 \times 0$
2	5	$3 + 2 = 3 + 2 \times 1$
3	7	$3 + 4 = 3 + 2 \times 2$
4	9	$3 + 6 = 3 + 2 \times 3$
5	11	$3 + 8 = 3 + 2 \times 4$
⋮		⋮
⋮		⋮
n		$3 + 2 \times (n-1)$

The n^{th} term $= 3 + 2 \times (n-1)$
 $= 3 + 2n - 2$

General term $= \underline{2n+1}$

Check that the value in this column is 1 less than the value in the serial number column

Example 3

Find the general term of the number pattern 35, 33, 31, 29, 27 ...

35, 33, 31, 29, 27 (The difference between the terms is -2)

$\underbrace{\quad\quad}_{-2}$ $\underbrace{\quad\quad}_{-2}$ $\underbrace{\quad\quad}_{-2}$ $\underbrace{\quad\quad}_{-2}$

Serial number of the term	Value of the term	Value of the term in terms of the first term
1	35	$35 = 35 - 2 \times 0$
2	33	$35 - 2 = 35 - 2 \times 1$
3	31	$35 - 4 = 35 - 2 \times 2$
4	29	$35 - 6 = 35 - 2 \times 3$
5	27	$35 - 8 = 35 - 2 \times 4$
⋮		⋮
⋮		⋮
n		$35 - 2 \times (n-1)$

The n^{th} term $= 35 - 2 \times (n-1)$
 $= 35 - 2n + 2$

General term $= \underline{37 - 2n}$

Check that the value in this column is 1 less than the value in the serial number column

Example 4

Find the general term of the number pattern 3, 9, 27, 81, ...

$$\begin{array}{cccc}
 3, & 9, & 27, & 81 \\
 \curvearrowright & \curvearrowright & \curvearrowright & \\
 \times 3 & \times 3 & \times 3 &
 \end{array}$$

Serial number of the term	Term	Value of the term in terms of the first term
1	3	3^1
2	9	3^2
3	27	3^3
4	81	3^4
⋮		⋮
⋮		⋮
⋮		⋮
n		3^n

Check that the serial number is same as the index in this column

The n^{th} term is 3^n

The general term = 3^n

Example 5

Find the general term of the number pattern 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, ...

$$\begin{array}{cccc}
 5, & 5\frac{1}{2}, & 6, & 6\frac{1}{2}, & 7 & (\frac{1}{2} \text{ is added}) \\
 \curvearrowright & \curvearrowright & \curvearrowright & \curvearrowright & \\
 +\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} &
 \end{array}$$

Serial number of the term	Term	Value of the term in terms of the first term
1	5	$5 + \frac{1}{2} \times 0$
2	$5\frac{1}{2}$	$5 + \frac{1}{2} \times 1$
3	6	$5 + \frac{1}{2} \times 2$
4	$6\frac{1}{2}$	$5 + \frac{1}{2} \times 3$
⋮		⋮
⋮		⋮
⋮		⋮
n		$5 + \frac{1}{2} \times (n-1)$

The value in this column is 1 less than the value in the serial number column

The n^{th} term = $5 + \frac{1}{2} \times (n - 1)$

\therefore The General term = $5 + \frac{1}{2} \times (n - 1)$

Exercise 2.3

(1) Write down the next three terms of each of the number patterns given below.

(i) 7, 10, 13, 16, ...

(ii) 100, 95, 90, 85, ...

(iii) $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots$

(iv) 5, 25, 125, 625, ...

(v) 2.25, 2.5, 2.75, 3, ...

(vi) -20, -10, 0, 10, ...

(vii) $\frac{1}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}, \dots$

(viii) 500, 50, 5, 0.5, ...

(2) Write down the general term relevant to each of the number patterns given below.

(i) 7, 14, 21, 28, ...

(ii) 3, 7, 11, 15, ...

(iii) 5, 12, 19, 26, ...

(iv) 7, 5, 3, 1, ...

(v) 2, 4, 8, 16, ...

(vi) -2, 4, -8, 16, ...

(vii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

(viii) $\frac{4}{1}, \frac{5}{2}, \frac{6}{3}, \frac{7}{4}, \dots$

(ix) $\frac{3}{1}, \frac{3}{4}, \frac{3}{9}, \frac{3}{16}, \dots$

(x) $(1 \times 4), (2 \times 5), (3 \times 6), (4 \times 7), \dots$

2.4 Calculations in relation to the general term

Example 6

The general term of a number pattern is $3n + 1$.

- (i) Write down its first four terms.
- (ii) Find its twenty first term.
- (iii) Which term is 151?
- (iv) Find its $(n+1)^{\text{th}}$ term.

(i) The general term = $3n + 1$

The first term (when $n = 1$) = $3 \times 1 + 1 = 4$

The second term (when $n = 2$) = $3 \times 2 + 1 = 7$

The third term (when $n = 3$) = $3 \times 3 + 1 = 10$

The fourth term (when $n = 4$) = $3 \times 4 + 1 = 13$

The first four terms are : 4, 7, 10, 13.

(ii) The 21^{st} term

(when $n = 21$):

= $3 \times 21 + 1$

= $63 + 1$

= 64

(iii) Let 151 be the n^{th} term
Solve the following equation and obtain the value of 'n'

$$3n + 1 = 151$$

$$3n = 151 - 1$$

$$3n = 150$$

$$\frac{3n}{3} = \frac{150}{3}$$

$$\underline{n = 50} \quad \therefore 151 \text{ is its } 50^{\text{th}} \text{ term.}$$

(iv) The n^{th} term = $3n+1$

$$\begin{aligned} \therefore (\text{By substituting } n+1 \text{ for } n) \\ \text{term } (n+1) &= 3(n+1) + 1 \\ &= 3n + 3 + 1 \\ &= \underline{\underline{3n + 4}} \end{aligned}$$

Example 7

A thin long wire is cut into pieces such that the first piece is 5 cm long and every other piece that is cut after that is 4 cm longer than the length of the preceding piece.

- Write down the lengths of the first four pieces separately.
- Obtain an expression for the length of the n^{th} piece.
- Find the length of the 10^{th} piece that is cut.
- Which piece is 85cm long?

(i) 5, 9, 13, 17

(ii) $5 + (n - 1) \times 4 = 5 + 4n - 4 = 4n + 1$

(iii) The length of the 10^{th} piece (when, $n = 10$)
 $= 4 \times 10 + 1$
 $= \underline{41 \text{ cm}}$

(iv) Let the n^{th} piece be of length 85 cm.

Then $4n + 1 = 85$

$$4n = 85 - 1$$

$$\frac{4n}{4} = \frac{84}{4}$$

$$n = 21$$

The length of the 21^{st} piece is 85 cm

Exercise 2.4

(1) Select from the expressions given below, the appropriate general term for each of the following number patterns and write it in the space provided.

$10n$

n^3

10^n

n^2

3^n

$n(n+1)$

(I) 1, 4, 9, 16, 25, -----

(ii) $1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, \dots$

(iii) 10, 100, 1 000, 10 000, ----

(iv) 1, 8, 27, 64, ----

(v) 10, 20, 30, 40, 50, ----

(vi) 3, 9, 27, 81, ----

(2) Write down the first four terms of the number patterns, which the following general terms represent.

(i) $5n + 1$ (ii) $2n - 2$ (iii) $(-3)^n$ (iv) $\frac{n+4}{n+1}$ (v) $\frac{5}{n}$ (vi) $1 + (-1)^n$

(3) Consider each of the number patterns given below and mention who has written the correct general term; only by substituting values for 'n'

	Number pattern	Deepal	Gunasiri	Sagara
i	1, 5, 9, 13, ----	$3n - 2$	$4n - 3$	$2n - 1$
ii	3, 8, 13, 18, ----	$5n - 2$	$2n + 1$	$n + 1$
iii	7, 9, 11, 13, 15, ---	$3n + 4$	$n + 6$	$2n + 5$

Can a general term be decided by considering only the first two or three terms?

(4) Find the n^{th} term of each of the following number patterns and find the term mentioned in front of each pattern.

(i) 7, 10, 13, 16, ---- 18^{th} term

(ii) 8, 10, 12, 14, ---- 10^{th} term

(iii) 32, 30, 28, 26, ---- 15^{th} term

(iv) 3, 8, 13, 18, ---- 30^{th} term

(v) 4, 16, 64, 256, ---- 7^{th} term

(vi) $\frac{2}{1}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \text{----}$ 8^{th} term

(5) ● ○ ● ● ○ ○ ● ● ○ ○ ○ ●

Shown above is a pattern formed by using white balls and black balls.

(i) Draw the next arrangement of the above pattern.

(ii) How many white and black balls will be there in the 7^{th} arrangement ?

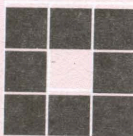
(iii) The general term of the above pattern is $n+2$. Considering the number of black balls and white balls, write down the colour of the balls represented by n and 2.

(iv) Using the general term of (iii) above, find the number of white balls and the total number of balls in the 15^{th} arrangement.

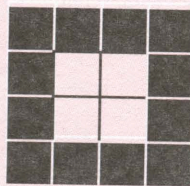
(v) If a pupil plans to form a pattern similar to the above with 20 balls, find how many white balls he should use.

(6) Sajitha collected Rs 15 in a till in the first week, Rs 20 in the second week and Rs 25 in the third week, with the intention of buying a book which costs Rs 750.

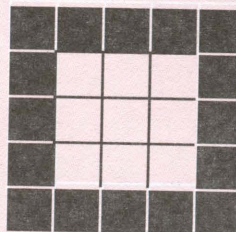
- (i) Write down the amount of money that she puts into the till during the first five weeks.
- (ii) Write down the general term of the number pattern according to which she is collecting money.
- (iii) Starting from the first week, in which week will she put Rs 85 into the till?
- (7) The initial annual salary of a person is Rs 23 000. This increases annually by Rs 3 000.
- (i) Write the monthly salary for the first three years.
- (ii) Obtain a general formula to calculate the salary in the n^{th} year.
- (iii) Using the above formula, find his monthly salary after 5 years of service.
- (iv) Starting from his first year of service, when will his annual salary be Rs 44 000?
- (8) Nadishani took some empty match boxes and numbered them as 1, 2, 3 etc. She put match sticks into these boxes such that each box contains 3 matchsticks more than the preceding box.
- (i) write the number of matchsticks in the first four boxes.
- (ii) obtain an expression in terms of 'n' to find the number of matchsticks in any box named.
- (iii) find the number of matchsticks in the 10^{th} box.
- (iv) what is the number written on the box which has 49 matchsticks?
- (9) The general term of a number pattern is $5 - 3n$.
- (i) Write down its first four terms.
- (ii) Find its 15^{th} term.
- (iii) Which term is -85 ?
- (iv) Show that there cannot be a term in this pattern whose value is -50
- (10) Shown below are the first three stages of a pattern planned to be made with black and rose coloured square floor tiles of the same size.



(i)



(ii)



(iii)

(i) Study the above pattern and find the value of P, Q, R of the table given below.

Number of tiles in a side	3	4	5	6	...	P	...	15	...	20
Total number of black coloured tiles	8	12	16	20	...	36	...	Q	...	76
Total number of rose coloured tiles	1	4	9	16	...	64	...	169	...	R

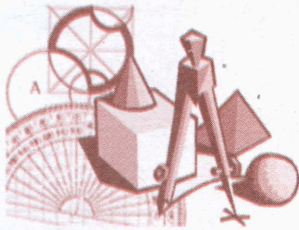
(ii) Calculate the following in terms of 'n' relevant to the pattern number 'n'

a - Total number of tiles needed

b - Total number of rose coloured tiles needed

c - The number of tiles needed for a side.

- (iii) (a) Find using (ii) above, the pattern number in which 124 black tiles are used.
- (b) Find the number of rose coloured tiles used and the number of tiles used for a side in the above stage.



Fractions

03

By studying this chapter you will be able to achieve the following competencies.

- ★ Simplifying fractions including 'brackets' and 'of'
- ★ Following the correct order when simplifying expressions with fractions.

3.1 Elementary mathematical operations on fractions

Let us recollect facts about simplifying fractions which we have studied earlier; by examples and exercises.

Example 1

Simplify the fractions given below and give the answers in the simplest form.

(Fractions with equal denominators)

$$(i) \quad \frac{1}{8} + \frac{5}{8}$$

$$= \frac{\cancel{6}^1}{\cancel{8}^2} = \underline{\underline{\frac{3}{4}}}$$

$$(ii) \quad \frac{5}{12} - \frac{3}{12}$$

$$= \frac{\cancel{2}^1}{\cancel{12}_6} = \underline{\underline{\frac{1}{6}}}$$

(Fractions with unequal denominators)

$$(iii) \quad \frac{3}{4} + \frac{2}{5}$$

$$= \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1 \frac{\underline{\underline{3}{20}}}$$

(Fractions with unequal denominators) (Mixed numbers)

$$(iv) \quad \frac{3}{7} - \frac{1}{3}$$

$$= \frac{9}{21} - \frac{7}{21}$$

$$= \underline{\underline{\frac{2}{21}}}$$

$$(v) \quad 2\frac{5}{8} - 1\frac{5}{12}$$

$$= (2-1) + \left(\frac{5}{8} - \frac{5}{12}\right)$$

$$= 1 + \left(\frac{15}{24} - \frac{10}{24}\right)$$

$$= 1 + \frac{5}{24}$$

$$= 1 \frac{\underline{\underline{5}{24}}}$$

Method (i)

$$(vi) \quad 3\frac{3}{4} - 2\frac{1}{12}$$

$$= \frac{15}{4} - \frac{25}{12}$$

$$= \frac{45}{12} - \frac{25}{12}$$

$$= \frac{20}{12} = \frac{5}{3}$$

$$= 1 \frac{\underline{\underline{2}{3}}}$$

Method (ii)

$$3 - 2 + \left(\frac{3}{4} - \frac{1}{12}\right)$$

$$= 1 + \frac{9}{12} - \frac{1}{12}$$

$$= 1 + \frac{\cancel{8}^2}{\cancel{12}_3}$$

$$= 1 \frac{\underline{\underline{2}{3}}}$$

Example 2

Simplify and give the answers in the simplest form

$$(i) \frac{2}{5} \times \frac{6}{7}$$

$$= \frac{12}{35}$$

$$(ii) \frac{12}{13} \times \frac{5}{8}$$

$$= \frac{12^3}{13} \times \frac{5}{8_2}$$

$$= \frac{15}{26}$$

$$(iii) 2\frac{2}{5} \div 6$$

$$= \frac{12}{5} \div 6$$

$$= \frac{12^2}{5} \times \frac{1}{6_1} = \frac{2}{5}$$

Exercise 3.1

(1) Simplify the fractions given below and give the answer in the simplest form.

$$(i) \frac{1}{5} + \frac{2}{5}$$

$$(ii) \frac{5}{7} - \frac{3}{7}$$

$$(iii) \frac{1}{4} + \frac{2}{8}$$

$$(iv) \frac{2}{5} - \frac{1}{10}$$

$$(v) 1\frac{3}{4} \times \frac{1}{7}$$

$$(vi) \frac{2}{8} \div \frac{3}{2}$$

$$(vii) 2\frac{5}{8} \div 1\frac{3}{4}$$

$$(viii) 3\frac{3}{4} \div 5$$

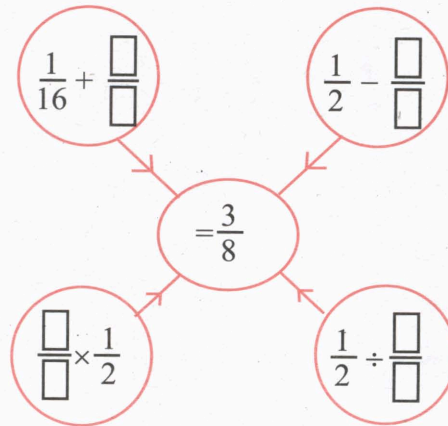
$$(ix) 2\frac{2}{3} \times 1\frac{4}{5}$$

(2) Copy and complete the following tables

+			$\frac{1}{3}$
$\frac{1}{2}$	$\frac{3}{4}$		$\frac{5}{6}$
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	
$\frac{1}{5}$			

×	$\frac{5}{2}$		
$\frac{1}{3}$		$\frac{1}{4}$	
		$\frac{3}{8}$	
$\frac{1}{4}$			$\frac{1}{6}$

(3) Find suitable numbers for the boxes such that the answer $\frac{3}{8}$ is obtained



3.2 Simplification of expressions including fractions with brackets.

Example 3

Simplify: $\left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{1}{5}$

There are two mathematical operations here, one of which is separated with brackets. The rule is to simplify first, that part which is within brackets. That is, the bracket indicates that particular part which should be simplified first.

Accordingly, $\left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{1}{5} = \left(\frac{3}{6} + \frac{2}{6}\right) \times \frac{1}{5}$ (The part within brackets first)

$$= \frac{5}{6} \times \frac{1}{5} = \underline{\underline{\frac{1}{6}}}$$

Example 4

Simplify $\left(\frac{1}{2} + \frac{1}{3}\right) \div \frac{1}{6}$ and give the answer in the simplest form.

$$\left(\frac{1}{2} + \frac{1}{3}\right) \div \frac{1}{6} = \left(\frac{3}{6} + \frac{2}{6}\right) \div \frac{1}{6}$$

(The part within brackets first)

$$= \frac{5}{6} \times \frac{6}{1} = \underline{\underline{5}}$$

Example 5

Simplify and give the answer in the simplest form: $\frac{1}{5} \times \left(\frac{2}{3} + \frac{1}{2}\right)$

$$\begin{aligned}\frac{1}{5} \times \left(\frac{2}{3} + \frac{1}{2}\right) &= \frac{1}{5} \times \left(\frac{4}{6} + \frac{3}{6}\right) \text{ (The part within brackets first)} \\ &= \frac{1}{5} \times \frac{7}{6} = \underline{\underline{\frac{7}{30}}}\end{aligned}$$

Example 6

Simplify and give the answer in the simplest form: $\left(2\frac{3}{5} - 1\frac{1}{4}\right) \div \frac{5}{8}$

$$\begin{aligned}&= \left(2\frac{3}{5} - 1\frac{1}{4}\right) \div \frac{5}{8} = \left(\frac{13}{5} - \frac{5}{4}\right) \div \frac{5}{8} \text{ (Simplify the part within brackets first)} \\ &= \left(\frac{52}{20} - \frac{25}{20}\right) \div \frac{5}{8} \\ &= \frac{27}{20} \times \frac{8}{5} = \frac{54}{25} = \underline{\underline{2\frac{4}{25}}}\end{aligned}$$

Example 7

What is the value obtained when the sum of the fractions $\frac{2}{3}$ and $\frac{1}{4}$ is multiplied by 24?

Here let us show by inserting brackets that simplification of the sum of the two fractions should be done first.

Accordingly it is,

$$\begin{aligned}\left(\frac{2}{3} + \frac{1}{4}\right) \times 24 &= \left(\frac{8+3}{12}\right) \times 24 \\ &= \frac{11}{12} \times 24 = \underline{\underline{22}}\end{aligned}$$



Exercise 3.2



Simplify the expressions with fractions given below and give the answer in the simplest form.

1. $\left(\frac{5}{6} + \frac{1}{8}\right) \div \frac{1}{12}$

2. $\left(1\frac{1}{9} + 1\frac{1}{2}\right) \div \frac{2}{9}$

3. $\frac{3}{5} \times \left(\frac{3}{8} - \frac{1}{6} \right)$

4. $\frac{1}{5} + \left(2\frac{1}{3} + 1\frac{1}{2} \right)$

5. $\left(2\frac{2}{3} + 1\frac{1}{4} \right) \div \frac{5}{6}$

6. $6\frac{1}{2} \times \left(\frac{1}{2} + \frac{1}{3} - \frac{3}{4} \right)$

3.3 Simplification of expressions with fractions including 'of'

Example 8

What is $\frac{1}{9}$ of 27 cm?

That is, $\frac{1}{9}$ of 27 cm

Here 'of' is considered as multiplication and simplified

$$= \frac{1}{9} \times 27$$

$$= \underline{\underline{3 \text{ cm}}}$$

Example 9

How many days are there in $\frac{3}{5}$ of a year?

Let us indicate the above with mathematical symbols and simplify.

That is, $\frac{3}{5}$ of 1 year

$$= \frac{3}{5} \text{ of } 365 \text{ days}$$

$$= \frac{3}{5} \times 365 \text{ days} = \underline{\underline{219 \text{ days}}}$$

Let us consider an example in which there are other mathematical operations along with 'of'

Example 10

Find the answer when $\frac{4}{7}$ of $4\frac{2}{3}$ is divided by $1\frac{1}{9}$

Let us indicate the above expression with mathematical symbols.

$$\frac{4}{7} \text{ of } 4\frac{2}{3} \div 1\frac{1}{9} = \frac{4}{7} \times \frac{14^2}{3} \div 1\frac{1}{9}$$

$$= \frac{8}{3} \div \frac{10}{9}$$

Here the rule is to consider of as multiplication and that part should be simplified first

$$= \frac{8^4}{7_1} \times \frac{9^3}{10_5} = \frac{12}{5} = \underline{\underline{2\frac{2}{5}}}$$



Exercise 3.3



1. Find :

(i) $\frac{2}{5}$ of 50 rupees (ii) $\frac{2}{3}$ of one minute in seconds (iii) $\frac{1}{3}$ of one hour in minutes

2. Simplify : (i) $\frac{5}{8}$ of $\frac{8}{5}$ (ii) $\frac{2}{5}$ of $\frac{2}{3} + \frac{1}{5}$ (iii) $\frac{2}{7} + \frac{2}{5}$ of $\frac{1}{3}$

3.4 The order in which mathematical operations should be performed in expressions of fractions.

The order in which expressions of fractions including brackets, 'of' and the four basic mathematical operations should be simplified is given below.

1. Simplification of the part within brackets (Brackets)
2. Simplification of the part connected with 'of' (Of)
3. Simplification of the part with division (Division)
4. Simplification of the part with multiplication (Multiplication)
5. Simplification of the part with addition (Addition)
6. Simplification of the part with subtraction (Subtraction)

Let us indicate the above order as " **BODMAS**" for easy reference. Let us examine how expressions with fractions are simplified using the above mentioned order through the examples given below.

◆ Note : This is the order in which operations on integers are also performed.

Example 11

Simplify and give the answer in the simplest form $\frac{2}{17}$ of $\left(\frac{1}{6} + \frac{2}{5}\right) \div \frac{3}{5}$

$$= \frac{2}{17} \text{ of } \left(\frac{1}{6} + \frac{2}{5} \right) \div \frac{3}{5}$$

$$= \frac{2}{17} \text{ of } \left(\frac{5+12}{30} \right) \div \frac{3}{5} \quad (\text{The part within brackets first})$$

$$= \frac{2}{17} \text{ of } \frac{17}{30} \div \frac{3}{5} \quad (\text{'of' - second})$$

$$= \frac{2^1}{17_1} \times \frac{17^1}{30_{15}} \div \frac{3}{5}$$

$$= \frac{1}{18_3} \times \frac{5^1}{3}$$

$$= \underline{\underline{\frac{1}{9}}}$$

Example 12

Simplify and give the answer in the simplest form: $1\frac{1}{3} + \frac{5}{6}$ of $\frac{2}{5}$

$$1\frac{1}{3} + \frac{5}{6} \text{ of } \frac{2}{5} = 1\frac{1}{3} + \frac{5^1}{6_3} \times \frac{2^1}{5_1}$$

$$= 1\frac{1}{3} + \frac{1}{3}$$

$$= 1\frac{2}{3}$$

If the above rule was not followed, we would not have obtained the correct answer. This can be verified by simplifying this example in the order of mathematical operations given in the problem.

$$1\frac{1}{3} + \frac{5}{6} \text{ of } \frac{2}{5}$$

$$= \frac{4}{3} + \frac{5}{6} \text{ of } \frac{2}{5}$$

Let us do the addition first without following the rule.

$$= \frac{4}{3} + \frac{5}{6} \text{ of } \frac{2}{5}$$

$$= \frac{8}{6} + \frac{5}{6} \text{ of } \frac{2}{5} = \frac{13}{6} \text{ of } \frac{2}{5}$$

Let us do 'of' now

$$= \frac{13}{6} \times \frac{2}{5}$$

$$= \frac{26}{30} \text{ This is an incorrect answer}$$

What should be added to $1\frac{1}{3}$ is $\frac{5}{6}$ of $\frac{2}{5}$ and not $\frac{5}{6}$. Hence this answer is incorrect.

Accordingly, the correct answer to problems involving the simplification of fractions can be obtained only by following the **BODMAS** rule.

Example 13

Simplify and give the answer in the simplest form $\frac{5}{8} \times 1\frac{1}{2} \div \frac{15}{16}$

$$\frac{5}{8} \times 1\frac{1}{2} \div \frac{15}{16}$$

$$= \frac{\cancel{5}^1}{\cancel{8}_1} \times \frac{\cancel{3}^1}{\cancel{2}_1} \times \frac{16^1}{15_1}$$

$$= 1$$

Example 14

Simplify and give the answer in the simplest form: $\frac{\frac{3}{4} + \frac{2}{3}}{\frac{5}{6}}$

$$\left(\frac{3}{4} + \frac{2}{3}\right) \div \frac{5}{6}$$

(The bracket first)

$$= \left(\frac{9}{12} + \frac{8}{12}\right) \div \frac{5}{6}$$

(Division next)

$$= \frac{17}{12} \times \frac{6}{5}$$

$$= \frac{17}{10}$$

$$= \underline{\underline{1\frac{7}{10}}}$$

Example 15

How many $\frac{1}{4}$ kg packets of sugar can be made from 100 kg of sugar?

$$\text{We can express the answer as, } = 100 \div \frac{1}{4}$$

$$= 100 \times \frac{4}{1}$$

$$= 400$$

$$\therefore \text{ Number of packets of sugar } = \underline{\underline{400}}$$

Example 16

Simplify and give the answer in the simplest form $\left(2\frac{1}{4} \div \frac{3}{14}\right) \times 2\frac{1}{7}$

$$\left(2\frac{1}{4} \div \frac{3}{14}\right) \times 2\frac{1}{7} = \left(\frac{9}{4} \div \frac{3}{14}\right) \times \frac{15}{7} \quad (\text{Expressing mixed numbers as improper fractions})$$

$$= \left(\frac{9^2}{4_2} \times \frac{14^7}{3_1}\right) \times \frac{15}{7} \quad (\text{The brackets next})$$

$$= \frac{21^3}{2} \times \frac{15}{7_1} = \frac{45}{2} = \underline{\underline{22\frac{1}{2}}}$$

Example 17

Simplify and give the answer in the simplest form: $\frac{4}{9} + 1\frac{1}{2} \div \frac{3}{5} - 2\frac{1}{3}$

$$\frac{4}{9} + 1\frac{1}{2} \div \frac{3}{5} - 2\frac{1}{3} = \frac{4}{9} + \frac{3}{2} \div \frac{3}{5} - \frac{7}{3} \quad (\text{Expressing as improper fractions})$$

$$= \frac{4}{9} + \frac{3^1}{2} \times \frac{5}{3_1} - \frac{7}{3} \quad (\text{Division})$$

$$= \frac{4}{9} + \frac{5}{2} - \frac{7}{3}$$

$$= \frac{8}{18} + \frac{45}{18} - \frac{7}{3} \quad (\text{Addition})$$

$$= \frac{53}{18} - \frac{42}{18} = \underline{\underline{\frac{11}{18}}}$$



Exercise 3.4



1. Simplify the following expressions involving fractions and give the answer in the simplest form.

(i) $\frac{1}{3} - \frac{1}{2} \times \frac{1}{4}$

(i) $\left(\frac{1}{2} - \frac{1}{5}\right) \div \frac{4}{7}$

(iii) $\frac{1}{2} - \frac{1}{5} \div \frac{4}{7}$

(iv) $\frac{3}{4} \div \frac{1}{2} + \frac{1}{8}$

(iv) $\frac{5}{8} \times 1\frac{1}{2} \div \frac{15}{16}$

(iv) $\frac{2}{5} \times \frac{9}{10} \div \frac{27}{40}$

(vii) $\left(\frac{3}{5} \div \frac{18}{55}\right) \times \frac{9}{11}$

(vii) $\frac{1}{2} \times \frac{3}{5} \div \frac{5}{9}$

(vii) $\left(\frac{3}{7} \div \frac{8}{21}\right) \times \frac{2}{5}$

(x) $\frac{14}{25} \times \frac{5}{9} \div \frac{7}{8}$

(x) $\left(\frac{3}{10} + \frac{2}{5}\right) \div \frac{7}{15}$

(xii) $\frac{\frac{1}{6} + \frac{1}{2}}{\frac{1}{5}}$

(xiii) $2\frac{1}{2} \times 2\frac{2}{5} \div \frac{3}{5}$

(xiv) $\left(3\frac{1}{3} \div 2\frac{1}{6}\right) \times \frac{1}{4}$

(xv) $\frac{4}{9} \div 1\frac{1}{2} \div \frac{3}{5} - 2\frac{1}{3}$

2. Insert the symbol (✓) in the box against each statement given below if correct and the symbol (×) if incorrect

(i) $3\frac{2}{3} \div 1 = 3\frac{2}{3} \times 1$

(ii) $\frac{3}{2} + \frac{1}{2} \div 2 = 1$

(iii) $4\frac{2}{3}$ of $\frac{4}{7} = 4\frac{2}{3} \times \frac{4}{7}$

(iv) $\frac{1}{2} + \left(1\frac{1}{2} - \frac{1}{2}\right) = \frac{1}{2}$ of 3

(v) $1\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = 1\frac{1}{5} - \frac{2}{7} \times \frac{1}{3}$

$$(vi) \frac{2}{13} \text{ of } \left(5\frac{1}{3} + 6\frac{2}{3}\right) = \left(7\frac{1}{2} - 3\frac{1}{3}\right) - 2\frac{1}{12}$$



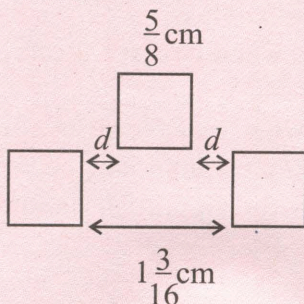
$$(vii) \frac{2}{13} \text{ of } 50 = \left(7\frac{1}{2} - 3\frac{1}{3}\right) - 2\frac{1}{12}$$



$$(viii) 1\frac{3}{5} + \frac{1}{6} \times \frac{3}{8} = \frac{2}{5} \text{ of } \left(7\frac{1}{3} - 5\frac{1}{2}\right)$$



3. Shown below is how three pieces of cardboard are placed on a table.



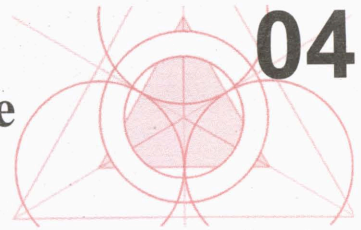
Calculate the length 'd',

4. (i) The thickness of a floor tile is $\frac{3}{8}$ cm. What is the minimum height of a box needed to pack 12 such tiles placing one over the other?
- (ii) How many floor tiles of the above thickness can be placed one over the other in a box of depth 12 cm?
5. A rubber ball is dropped on to a flat floor from a height of 300 cm. Each time it bounces after touching the floor, the ball comes up $\frac{4}{5}$ of the earlier height.
- (i) Find the height it reaches during the first bounce
- (ii) Find the height it reaches in the second bounce.
6. One half of a container with a capacity of 5l holds water and the remaining half is filled with oil. Two thirds of a second container of identical capacity is filled with water and one third with oil. The contents of the two containers are poured into another container with a capacity of 10l. Find what fraction of this container is,
- (i) filled with water
- (ii) filled with oil.



Percentage

04



By studying this chapter you will be able to achieve the following competencies

- ★ Calculating the profit gained or loss incurred in running a business.
- ★ Deciding on the more advantages transaction.
- ★ Calculating the third quantity, when two are known out of the cost price, selling price and profit or loss.
- ★ Finding the third value, when two are given out of, discount, marked price and selling price.
- ★ Finding the third value, when two are given out of the selling price, percentage commission and commission.
- ★ Performing business with an understanding of discounts and commissions.

4.1 Profit and loss

I will pay Rs 3 for a fruit.
Let me have the whole
stock of Rambutan.



Father sells a Rambutan
fruit at Rs 5 each. Then
we get a profit of Rs 2
Per fruit



A trader always expects to sell goods at a price above the price he buys them at. When he is able to do so we say that he makes a **profit**.

There are occasions when goods become rotten or damaged. During such times the trader has to sell the goods to a price below the price he bought them at. In a situation like this, we say that he **losses** on the deal.

A trader buys a Pineapple fruit for Rs 45 and sells it at Rs 75. He also buys a Papaw fruit for Rs 25 and sells it at Rs 55.

Here the profit he makes by selling the pineapple is Rs 30. The profit he makes by selling the papaw is also Rs 30. Though he makes an equal profit from both, he has spent a lesser amount of money to buy the papaw. Hence in this case selling papaw is more profitable than selling pineapple.

By finding the difference between the selling price and the cost price a quantitative profit or loss can be obtained.

Exercise 4.1

1. An incomplete table with the cost price and the selling price of certain goods is given below.

Fill in the blanks in the table

Category	Cost price (Rs)	Selling price (Rs)	Whether a profit/ loss	Profit/ loss in Rupees
(i) A book	80	115	-----	-----
(ii) A pen	-----	12.50	Profit	1.50
(iii) A childrens' shirt	125	115	-----	-----
(iv) A pineapple fruit	-----	55	Loss	5
(v) A bicycle	4 400	-----	Profit	900
(vi) A cup and a saucer	-----	275	Loss	25

2. Given below are three sets of transactions. Write with reasons which transaction is more profitable.

- (i) (a) A childrens' shirt is bought for Rs 150 and sold at Rs 210
 (b) A childrens' frock bought for Rs 150 is sold at Rs 225
- (ii) (a) Buying 1kg of beans for Rs 50 and selling it at Rs 80
 (b) Buying 1kg of carrot for Rs 60 and selling it at Rs 90
- (iii) (a) Selling an almirah for Rs 15 000 of which the cost of production is Rs 9 500
 (b) Selling a table for Rs 13 500 of which the cost of production is Rs 8 000

3. A trader bought 100 mangoes at Rs 12 each, out of which 8 were rotten and were thrown away. The rest of the mangoes were sold at Rs 20 each. Find the profit he made or the loss he had to incur.

4.2 Percentage of profit / loss

Consider buying an orange for Rs 25 and selling it at Rs 40. Also consider buying a guava fruit for Rs 20 and selling it at Rs 33.

The profit in selling oranges = Rs 15

The profit in selling guava = Rs 13

Here neither the cost prices nor the profits are equal. As such it is not possible to decide on the more profitable business. Hence let us assume that the cost price is Rs 100 and consider the profit from the sale of each kind.

		Buying price (Rs)	Profit (Rs)
Oranges	1 fruit	25	15
	4 fruits	100	60
Guava	1 fruit	20	13
	5 fruits	100	65

Accordingly,

the profit made by selling oranges bought for Rs 100 is Rs 60. It can be shown as

$$\frac{60}{100} \text{ or } 60\%$$

The profit made by selling guava bought for Rs 100 is Rs 65. It can be shown as

$$\frac{65}{100} \text{ or } 65\%$$

As $65\% > 60\%$,

it is more profitable to sell guava.

As shown above, when the cost price or the buying price of an item is 100, the profit made by selling it is the percentage profit.

You have learnt earlier how to express a given fraction as a percentage. Accordingly, by showing a profit or a loss as a fraction of the cost price and converting it into a percentage, the percentage profit or loss can be calculated.

Example 1

Find the profit and the percentage profit obtained by selling 1kg of rice at Rs 75, which was bought for Rs 60.

$$\begin{aligned} \text{The profit} &= \text{Rs } 75 - \text{Rs } 60 \\ &= \underline{\underline{\text{Rs } 15}} \end{aligned}$$

$$\begin{aligned} \text{The percentage profit} &= \frac{15}{60} \times 100\% \\ &= \frac{15^1}{\cancel{60}^{\cancel{4}_1}} \times 100\% \\ &= \underline{\underline{25\%}} \end{aligned}$$

Example 2

Find the price at which a trader will sell a clock which he bought for Rs 300 keeping a profit of 15%

First method

Cost price (Rs)	Profit (Rs)	Selling price (Rs)
100	15	115
200	30	230
300	45	345

Second method

$$\begin{aligned} \text{The profit} &= \text{Rs } 300 \times \frac{15}{100} \\ &= \text{Rs } 45 \end{aligned}$$

(Cost price (Rs)	Profit (Rs))
100	↔	15	
300	↔	?	

$$\therefore \text{The selling price} = \text{Rs } (300 + 45) = \underline{\underline{\text{Rs } 345}}$$

$$\therefore \text{The selling price} = \underline{\underline{\text{Rs } 345}}$$

Third method

As the profit is 15% a clock bought for Rs 100 is sold for Rs 115

$$\begin{aligned} \therefore \text{The selling price} &= \text{Rs } 300 \times \frac{115}{100} \\ &= \underline{\underline{\text{Rs } 345}} \end{aligned}$$

(Cost price	Selling price)
100	↔	115	
300	↔	?	

Example 3

The cost of production of an article is Rs 750. Due to a defect in the item it has to be sold at a loss of 2%. Find the selling price

First Method

$$\begin{aligned} \text{The loss} &= \text{Rs } 750 \times \frac{2}{100} \\ &= \text{Rs } 15 \\ \therefore \text{The selling price} &= \text{Rs } 750 - \text{Rs } 15 \\ &= \underline{\underline{\text{Rs } 735}} \end{aligned}$$

(Cost price (Rs)	Loss (Rs))
100	↔	2	
750	↔	?	

- (3) A trader sells an electrical appliance for Rs1500, which he had bought for Rs1 200. Find his percentage profit.
- (4) The table given below shows the cost of production and the selling price of four products of a ceramic factory;

Product	Cost of production (Rs)	Selling Price (Rs)
A flower - vase	200	280
A wall decoration	175	230
A jug	150	210
A cup	40	55

- (i) Find separately the profit made and the percentage profit obtained by selling each production.
- (ii) Which product is most profitable? Give reasons for the answer.
- (5) A trader bought a stock of eggs for Rs 900. Some eggs were damaged and were discarded, the rest were sold. He had to bear a loss of 3% on the transaction. Find the amount of money he received by selling the eggs.
- (6) The table given below shows the type of vegetable, the cost of 1kg of each, and the percentage profit expected by selling, which a vegetable trader has planned.

Find separately the selling price of 1kg of each vegetable, so that he will get the expected percentage profit.

Type of vegetable	The buying price of 1kg (Rs)	Expected percentage profit
Beans	55	20%
Carrot	70	30%
Brinjal	45	12%
Pumpkin	40	15%

- (7) The selling price of a few articles in a shop and the percentage profit obtained after sales are shown in the table given below.

Article	Selling Price (Rs)	Percentage profit
A clock	2 400	20%
A tea set	580	30%
An electric kettle	1 200	15%
A thermos flask	558	24%

Find separately the buying price of each of the above articles.

- (8) The cost of production of a packet of milk powder of weight 400 g of a certain brand is Rs 180.
- Find the price the producer sells it at to a retail dealer if he keeps a profit of 20%
 - Find the price the retail dealer sells it to a consumer at if he keeps a profit of 15%
 - Find how much more a consumer spends on a packet of milk than its cost of production.

4.3 Discount and Commission

Tharaka - Brother Saman, I bought a book yesterday, Its price was Rs 140, but the trader took only Rs 119 from me. He said a discount is allowed for the book. What is this discount brother?

Saman - Brother, a discount is an amount of money that is reduced from the marked price, when selling. A discount is usually expressed as a percentage of the marked price of the article.

Tharaka - Brother, then the trader will lose.

Saman - No brother, they mark the price keeping a profit. This amount is reduced from that profit.

Listen to me, they buy a book for Rs 80 and mark the price as Rs.140. When selling, a discount is given and the book is sold at Rs 119. Don't you see, there is still a profit.

Tharaka - Also brother, more people come to the shop, when discounts are given. Then there will be more sales and more profit.

Saman - You are correct. Now I see that you can understand.

Tharaka - My father, sold a block of land recently. He came home and told my mother "the commission was Rs 25 000." Brother, is this commission too similar to a discount?

Saman - No brother, when we want to sell things like a block of land, a vehicle

or a house, finding a buyer is rather difficult. We also have problems with time. In a situation like this, there are people who find buyers. They are called brokers. When we sell something with the help of a broker we will have to pay him for the help he has given. That is known as 'commission'.

Here your father has paid Rs 25 000 to the broker as commission from the money he obtained by selling the land.

Tharaka - Thank you brother for explaining all that to me.

Example 5

A trader gives a 5% discount when selling an article, the marked price Rs 3400

(i) What is the discount allowed?

(ii) What is the selling price after giving the discount?

$$\begin{aligned} \text{(i) The discount} &= \text{Rs } 3400 \times \frac{5}{100} \\ &= \underline{\underline{\text{Rs } 170}} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \therefore \text{ The selling price} &= \text{Rs } 3400 - 170 \\ &= \underline{\underline{\text{Rs } 3230}} \end{aligned} \quad \begin{pmatrix} \text{Discount} & \text{Marked} & \text{Selling} \\ \text{(Rs)} & \text{price (Rs)} & \text{price (Rs)} \\ 5 & \begin{matrix} \swarrow & \searrow \\ 100 & 95 \end{matrix} & \begin{matrix} \swarrow & \searrow \\ 3400 & ? \end{matrix} \\ ? & & ? \end{pmatrix}$$

(ii) Another method

As the discount is 5%, an article priced at Rs 100 is sold for Rs 95.

$$\begin{aligned} \therefore \text{ The selling price} &= \frac{95}{100} \times 3400 \\ &= \underline{\underline{\text{Rs } 3230}} \end{aligned}$$

$$\text{The discount} = \text{Rs } 3400 - \text{Rs } 3230 = \underline{\underline{\text{Rs } 170}}$$

Example 6

An article marked to be sold for Rs 8 500 is sold at Rs 8 075. Find the percentage discount given.

$$\begin{aligned} \text{The discount} &= \text{Rs } 8500 - \text{Rs } 8075 \\ &= \text{Rs } 425 \end{aligned}$$

$$\begin{aligned} \text{The percentage discount} &= \frac{425}{8500} \times 100\% \\ &= \underline{\underline{5\%}} \end{aligned}$$

Example 7

A broker charges a 3% commission on the selling price for selling a certain land. On a land sold for Rs 750 000

- (i) what is the commission he obtains?
(ii) what sum of money does the owner of the land obtain?

$$(i) \text{ The commission} = \text{Rs } 750\,000 \times \frac{3}{100} \quad \left(\begin{array}{cc} \text{Commission} & \text{Selling price} \\ \text{(Rs)} & \text{(Rs)} \\ 3 & \swarrow \searrow \\ ? & \swarrow \searrow \\ & 100 \\ & 750\,000 \end{array} \right)$$
$$= \underline{\underline{\text{Rs } 22\,500}}$$

$$(ii) \text{ The sum of money the owner obtains} \\ = \text{Rs } 750\,000 - 22\,500 \\ = \underline{\underline{\text{Rs } 727\,500.00}}$$

Example 8

A sales agent of a certain kind of biscuits gets a 5% commission on the value of the sales that he does. In a certain month he received Rs 18 300 as commission. What is the value of the sales he did during that month?

The 5% value he got from the sale of biscuits = Rs 18 300

$$\text{The value of the sale of biscuits} = 18\,300 \times \frac{100}{5} \quad \left(\begin{array}{cc} \text{Commission} & \text{Selling} \\ \text{(Rs)} & \text{(Rs)} \\ 3 & \swarrow \searrow \\ 18\,300 & \swarrow \searrow \\ & 100 \\ & ? \end{array} \right)$$
$$= \underline{\underline{\text{Rs } 366\,000}}$$

Another method

If the value of the sale of biscuits is Rs x ,

$$\text{the commission he received is} \quad = x \times \frac{5}{100} = 18\,300$$

$$\therefore x = 18\,300 \times \frac{100}{5}$$

$$\text{The value of the sale of biscuits} = \underline{\underline{\text{Rs } 366\,000}}$$

Exercise 4.3

(1) The marked price of a refrigerator is Rs 28 000. When selling, a discount of 10% is allowed.

- (i) What is the discount given in rupees?
(ii) What is the selling price of the refrigerator?

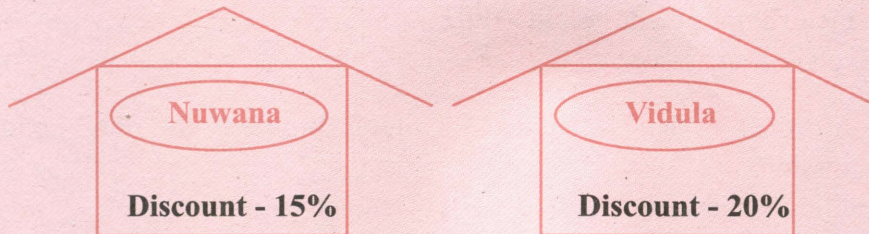
2.

SALE ! SALE! SALE!
25% Discount
On all ready made garments

Shown above is a board displayed in front of a shop during a festive season. From this shop, Sujith brought a shirt priced at Rs 350, two shirts priced at Rs 700 each and a trouser priced at Rs 1 300. Find,

- (i) her savings due to the discount
- (ii) the amount of money she had to pay.

3.



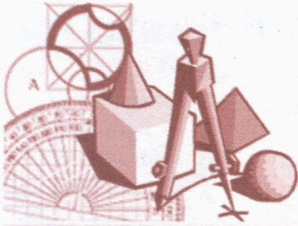
The marked price of a certain book is Rs 1 200 in both shops.

If a buyer, buys this book from,

- (i) Nuwana book shop, find his savings due to the discount.
 - (ii) Vidula book shop, find his savings due to the discount.
 - (iii) From which book shop should he buy this book to save more ?
 - (iv) What relationship is seen between percentage discount and the answer for (iii) above?
 - (v) If Vidula book shop had written the marked price of the book as Rs. 1300, then from which book shop would it be more profitable to buy the book?
- (4) A trader buys an article and marks its price expecting a profit of 25%. A 4% discount is given on the marked price for an outright purchase. if he sells it outright for Rs 840, find
- (i) its marked price.
 - (ii) its buying price.
- (5) Given below are the particulars about discounts which the book shop 'Mihira' allows for buyers when buying books in September, the month of Literature. For buying books to the value of
- Rs 5 000 or more a discount of 30%
 - Rs 3 000 up to Rs 5 000 a discount of 25%
 - Rs 1 000 up to 3 000 a discount of 20%
 - less than Rs 1 000 a discount of 15%

During this season, Nimesha bought books to the value of Rs 880, Tharaka to the value of Rs 3 050 and Lakshika to the value of Rs 2 900.

- (i) Find separately each persons savings.
 - (ii) Find separately the amount of money each had to pay.
 - (iii) Tharaka says that though he bought books of a higher value, it is Lakshika who has made a higher payment.
Do you agree with the above statement? Give your reasons.
 - (iv) If Lakshika added one more book of value Rs 100, will the above statement which Tharaka made be still valid?
Explain your answer.
- (6) An auctioneer of lands charges 3% of the proceeds of the auction as commission.
By selling a land for Rs 800 000 find the amount of money,
- (i) the auctioneer gets
 - (ii) the land owner gets
- (7) A broker was paid Rs 48 000 for selling a vehicle for Rs 1 200 000. What is the percentage commission he received?
- (8) A broker came to an agreement to sell a house on condition that he would be paid 5% of the sale as commission. If he received Rs 35 000 as commission, find the selling price of the house.



Simple Interest

05

By studying this chapter you will be able to achieve the following competencies.

- ★ Explaining the terms, Principal, Amount and Rate of Interest.
- ★ Calculating the interest, relevant to a given annual or monthly rate of interest, a given principal and a given time period.
- ★ Finding the rate of interest, when the principal and the relevant interest are given.
- ★ Finding the time, when the principal and the interest paid for a certain period of time are known.
- ★ Finding the principal when the interest for a given time period and a given rate of interest are known.
- ★ Making decisions after comparing interests.

5.1 Interest

18% Annual Rate of Interest
for you
who deposit money in our Bank
Deposit today in the 'Yasa Isuru'
account and receive this
attractive interest

Are you building a house?
Are you looking for a loan?
Come to us. We have
reduced the annual rate
of interest to 20%.

Pay attention to the two advertisements shown above.

The first advertisement gives information about the INTEREST a depositor receives.

The financial benefit a depositor gets from a bank annually or monthly is called INTEREST.

When money is deposited the depositor gets an interest.

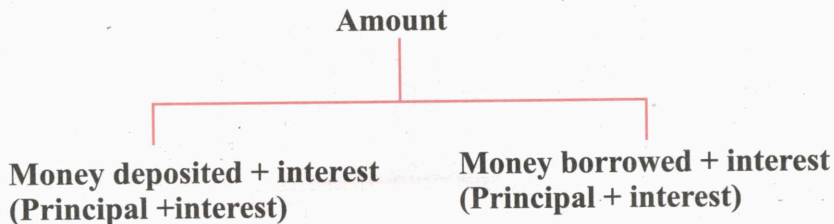
The second advertisement gives information about the INTEREST a person who obtains a loan has to pay.

After getting a loan, the additional sum of money the borrower has to pay the lender for giving the loan is called INTEREST. For a loan the borrower has to pay an interest to the lender.

Let us now get to know some other technical terms you will come across in this lesson.

A person who obtains a loan of Rs 25 000 at 18% simple interest per year pays back Rs 34 000 and gets released from the loan after 2 Years.

- ★ Here the Rs 25 000 loan obtained is called the **principal**.
- ★ A principal is either a sum of money taken as a loan or a sum of money deposited in a bank. Here Rs 34 000 is called the **amount**. **Amount** is the sum of the principal and the interest.



When Rs 25 000, the money borrowed, is deducted from the amount Rs 34 000 the difference is the interest paid.

When depositing money,
Interest received = Amount - money deposited (principal)

When borrowing money,
Interest paid = Amount - money borrowed (principal)

Rate of interest

18% simple interest for an year for deposits.

This means a deposit of Rs 100 will get Rs 18 as interest for a year

An Interest of 3% for a month for loans.

This means an interest of Rs 3 should be paid for a month for a loan of Rs100.

- ★ Interest is calculated based on the principal
- ★ Interest rate is written as a percentage.
- ★ Calculation of interest will either be annually or monthly and some times daily.

5.2 Calculating Simple Interest

Example 1

- (1) A person who borrowed Rs 25 000 at 20% simple interest per year gets released from of the loan after one year.

- (i) What is the interest he has to pay for that year?
 (ii) What is the amount he has to pay to get released from the loan?

(i) Interest for Rs 100 for 1 year = Rs 20

$$\begin{aligned} \therefore \text{The interest for Rs 25 000 for 1 year} &= \text{Rs } \frac{20}{100} \times 25000 \\ &= \underline{\underline{\text{Rs 5 000}}} \end{aligned}$$

(ii) The amount he should pay to get released from the loan = Rs 25 000 + 5 000
 = Rs 30 000

Example 2

(1) A person who borrowed Rs 15 000 at 16% annual simple interest pays back the loan with the interest after 3 years.

- (i) Find the interest he has to pay for the 3 years.
 (ii) What is the amount he has to pay to get released from the loan?

(i) The interest for Rs 100 for 1 year = Rs 16

$$\begin{aligned} \text{The interest for Rs 15 000 for 1 year} &= \text{Rs } \frac{16}{100} \times 15\,000 \\ &= \text{Rs 2 400} \end{aligned}$$

$$\begin{aligned} \text{The interest for Rs 15 000 for 3 years} &= \text{Rs } 2\,400 \times 3 \\ &= \underline{\underline{\text{Rs 7 200}}} \end{aligned}$$

(ii) The amount that has to be paid to released from the loan
 = Rs 15 000 + 7 200
 = Rs 22 200

Accordingly when the interest that should be paid for one year is calculated; the interest that should be paid for any number of years can be calculated. The interest calculated as above is known as simple interest.

When calculating simple interest for deposits and loans, three factors are taken into consideration.

- (i) **The amount of money - loan or deposit**
 (ii) **Time - time taken to pay back the loan or the period of deposit.**
 (iii) **The rate of interest.**



Exercise 5.1



- (1) A person who borrowed Rs 5 000 got released from the loan paying Rs 5 750 after one year. What is the interest he paid?

- (2) A person who borrowed Rs 24 000, paid back the loan during one year in 12 equal instalments each of Rs 2 310. What is the total interest he paid?
- (3) A person who borrowed Rs 30 000 at simple interest paid the amount Rs 40 800 after 3 years and was released from the loan.
 - (i) What is the interest he has paid for the 3 years?
 - (ii) Accordingly, what is the interest he has paid for one year?
- (4) Calculate the interest a person who borrowed Rs 40 000 at 18% simple interest per year has to pay for one year .
- (5) Nimali, who has deposited Rs 25 000 in a finance company, which pays 24% simple interest per year, withdraws the deposit along with the interest after a year
 - (i) What is the interest she receives for that year?
 - (ii) What is the amount she obtained at the end of the year?
- (6) Sithumini who pawned her gold jewellery in a pawnshop obtained Rs 30 000 and redeemed them after one year. The shop levies an annual interest of 24%. Find the amount she had to pay to redeem the pawned items?
- (7) Dileepa, who opened a fixed deposit account for Rs 75 000 in a financial institute which pays 18% simple interest per year, withdrew his money along with the interest after one year. What is the amount he received?
- (8) A businessman borrowed Rs 20 000 at 5% simple interest per month on condition that he will pay the relevant interest at the end of month and that the principal will be paid back only after the business develops.
 - (i) What is the interest he pays at the end of each month?
 - (ii) What is the total interest he pays at the end of 18 months?
 - (iii) If he could pay back the borrowed money after 18 months what is the amount he had to pay?
- (9) Sirimal deposited Rs 75 000 in a bank which pays 18% per year for fixed deposits. He received the relevant interest monthly from the bank. How much does he get from the bank as interest each month?
- (10) Sunimal who obtained a loan of Rs 30 000 from a money lender, for an annual simple interest at 20% got released from the loan after 3 years by paying back the money borrowed along with the interest. What is the amount he had to pay?
- (11) The credit society "Sanasa" lends money to its members at 8% simple interest per year. The loan with the interest can be paid in equal instalments. Given below is a quotation from a register of the society which shows information about loans obtained by 5 members. Fill in the blanks.

Name	Loan obtained (Rs)	Period agreed for payment	Total interest to be paid (Rs)	Sum of money to be paid (Rs)	Number of instalments to be paid as in agreement	Value of an instalment with interest
Kasun	15 000	2 years	24
Deleepa	36 000	3 years	36
Kumuditha	18 000	$1\frac{1}{2}$ years	18
alitha	30 000	$2\frac{1}{2}$ years	30
Sithumini	12 000	6 months	6

(12) Nimal, a government servant who received an arrears of his salary of Rs 120 000 deposited Rs 50 000 for one year as a fixed deposit in a bank which pays an interest of 18% per year. He bought a land for the balance Rs 70 000.

- What is the interest he will receive from the bank after one year?
- When the value of the land increased after one year by 20%, he sold the land. What is the profit he made from it?
- He could have bought another block of land with the money he deposited in the bank. How much did he lose or gain by not doing so?

(13) **12% simple interest will be paid annually for fixed deposits over Rs. 50 000**

Shown above is a part of an advertisement of a private financial organisation

- If only deposits of multiples of 1 000 can be made, what is the minimum deposit that should be made to get 12% simple interest?
- What will be the interest received if a person deposited Rs 80 000, for an year?
- Show that for a sum of money Rs p for a period of 't' years a person will get

$$p \left[1 + \frac{3t}{25} \right] \text{ as the amount after 't' years } (p > 50000).$$

(14)

3% monthly interest

24% annual interest

Shown above are two advertisements about the rates of interest of two financial institutions. It is necessary to select one institution out of the above two to deposit the monthly collection of Rs 48 000 of a welfare society.

- (i) Which financial institution will it be more profitable to select? Give reasons for your answer.
- (ii) What is the amount the welfare society will gain if the money is deposited in the institution that you decided on?

5.3 Calculating the annual rate of interest

Example 3

A man who borrowed Rs 30 000, paid back Rs 44 400 as the amount after 4 years and was released from the loan. Find the rate of interest charged for the loan.

$$\begin{aligned} \text{The interest paid for Rs 30 000 for 4 years} &= \text{Rs } 44\,400 - 30\,000 \\ &= \text{Rs } 14\,400 \\ \therefore \text{The interest paid for Rs 30 000 for 1 year} &= \text{Rs } \frac{14\,400}{4} \\ &= \text{Rs } 3\,600 \end{aligned}$$

$$\begin{aligned} \text{The interest for 1 year for Rs 100} &= \frac{3600}{30000} \times 100 \\ &= 12 \end{aligned}$$

$$\therefore \text{The annual rate of interest} = \underline{\underline{12\%}}$$

Exercise 5.2

1. Find the rate of interest in each of the following situations.

(a)	Principal	Time	Interest	Annual rate of interest
(i)	Rs 200	1 year	Rs 36	-----
(ii)	Rs 50	1 year	Rs 06	-----
(iii)	Rs 400	3 years	Rs 96	-----
(iv)	Rs 3 000	1 year 6 months	Rs 720	-----
(v)	Rs 15 000	1 year	Rs 1125	-----
(b)	Principal	Time	Interest	Monthly rate of interest
(i)	Rs 5 000	6 Years	Rs 2 700	-----
(ii)	Rs 10	1 Year	Rs 3	-----
(iii)	Rs 50	2 Years	Rs 8	-----

2. A person who borrowed Rs 25 000 had to pay an interest of Rs 7 000 for 2 years to be released from the loan. Find the rate of interest the money lender has charged.
3. Piyaseeli pawned her gold jewellery in a pawnshop and obtained Rs 36 000 and went abroad for three years. After returning home, she immediately redeemed her jewellery for which she had to pay Rs 58 680. Find the rate of interest of the pawnshop.
4. A person who borrowed Rs 200 000 from a bank at 24% interest per year lent it personally to a second person who was in need of a loan.
 - (i) What is the interest the first person has to pay to the bank?
 - (ii) This person expects to make a profit of Rs 24 000 during one year. To get the expected profit, what should be the rate of interest he should levy for the loan given to the second person?
5. Complete the table given below.

Statement showing interest	Rate of interest		
	Monthly	Annually	Quarterly
(i) Rs 3 for Rs 50 for 2 months	-----	-----	-----
(ii) Rs 36 for Rs 300 for 6 months	-----	-----	-----
(iii) Rs 3240 for Rs 18 000 for 18 months	-----	-----	-----
(iv) Rs 14 400 for Rs 48 000 for $2\frac{1}{2}$ years	-----	-----	-----

6. Kapila borrowed Rs 12 000 at 12% simple interest per year. After 4 months he borrowed another Rs 8 000 at the same rate of interest. After one year from the date of the first loan, he was released from both loans by paying back Rs 22 240.
 - (i) Calculate the interest he had to pay for the Rs 12 000.
 - (ii) What sum of money had he to pay to be released from the first loan?
 - (iii) What is the amount he had to pay to be released from the second loan?

5.4 Finding the time for which a loan is granted

Example 4

A person who obtained a loan of Rs 75 000 at 21% simple interest per year, paid Rs 47 250 as interest for a certain period of time. Find the period of time of the loan.

The loan obtained	= Rs 75 000
Interest for a year for Rs 75 000	= Rs 75000 \times $\frac{21}{100}$
	= Rs 15 750
Total interest paid	= Rs 47 250
The period of time of the loan	= $\frac{47250}{15750}$
	= <u>3 Years.</u>

Accordingly,

$$\text{Time} = \frac{\text{Total interest}}{\text{Annual interest}}$$

Exercise 5.3

1. What is the period of time for which the interest will be Rs 4 000 for a loan of Rs 25 000 at 8% annual simple interest?
2. A man who borrowed Rs 50 000 at 11% simple interest per year paid back Rs 66 500 after a certain period of time and was released from the loan. After what period of time did he get released?
3. By lending Rs 60 000 at $7\frac{1}{2}$ % simple interest per year a sum of Rs 4 500 is charged as interest. What is the period of time for which the money was lent?
4. A money lender who lends money at 2% simple interest for a month, has lent Rs 240 000. After how many months will he be able to earn Rs 38 400 as interest?
5. A private money lender has obtained a loan of Rs 150 000 from a commercial bank at 18% simple interest per year and has lent that money at 3% interest for a month.
 - (i) What is the interest he has to pay the commercial bank for a year?
 - (ii) What is the interest he receives in one month by lending that money?
 - (iii) What is the period of time necessary for him to earn the sum of money needed to pay the bank as interest?
 - (iv) If he is to earn the money needed to be paid as interest to the bank in 3 months, at what rate of interest should he lend the money?

5.5 Finding the Principal

When obtaining a loan or depositing money, we have to decide in advance about the possibility of paying the monthly instalment and the expected monthly interest from the

deposit. During such situations we should be able to find the principal when the relevant information is given.

Exercise 5

A finance company earns Rs 224 000 as interest in a year by lending money at 16% simple interest per year. Find the amount of money the company has lent.

The sum of money that should be lent to earn Rs 16 as
interest in a year = Rs 100

The sum of money lent to get Rs 224 000 as interest = $\frac{100}{16} \times 224000$
= Rs 1 400 000

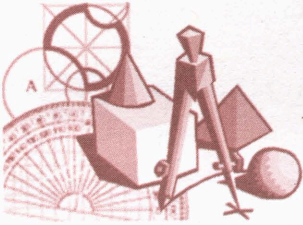
Exercise 5.4

1. A welfare society pays its members annually a simple interest of 12% on the available deposit on 31st December. Last year Sujeewa received Rs 168 as interest. What would have been the deposit in his name?
2. A businessman obtained a sum of money with an agreement of paying it back within a month with an interest of 4% a month. However he had misplaced the relevant document. But according to the receipt he had, the interest paid was Rs 1800. What sum of money had he borrowed?
3. A finance company pays an annual interest of 13% for deposits while charging 20% for loans. A part from the annual balance sheet of the company is shown below.
(The company calculates interest for deposits at the end of each year)

Total amount of money released as loans	=	<u> </u>
The interest received from loans	=	Rs 280 000
Total amount of money as deposits	=	<u> </u>
Interest paid for deposits	=	Rs 195 000
The annual profit	=	<u> </u>

Find the three sums of money unseen above due to blotches.

4. An interest of Rs 12 600 was paid for 3 years for a loan obtained at 12% simple interest per year. Find the loan obtained.
5. A person who has obtained a loan for 2 years at 15% annual interest has paid Rs 14 400 as interest. Find the sum of money he obtained as the loan.



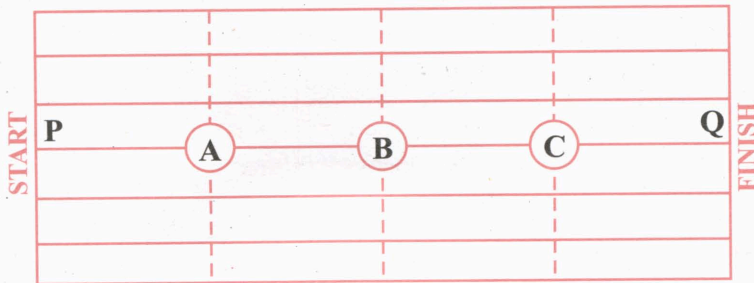
Algebraic Expressions

06

By studying this chapter you will be able to achieve the following competencies

- ★ Substituting in and simplifying algebraic expressions without indices and roots.
- ★ Obtaining the product of two binomial expressions.

6.1 Substitution



The above diagram shows how the field was prepared for the Mathematics day competition of the grade 8 pupils of Gunaratana Vidyalaya. Six competitors should start from P and finish at Q. The first three pupils to reach Q are the winners. The competitors should adhere to the following conditions.

- ★ Should select at random an algebraic expression of x from the box at A
- ★ Should select at random a value for x from the box at B
- ★ Should substitute the value for the expression at C
- ★ Should explain at Q how the answer was obtained.

Sujith who won the first place from the house "Number" presented his explanation as follows.

"The expression obtained at A is $4x - 3$. The value for x received at B is 2. As $4x = 4 \times x$, the expression obtained by substituting $x = 2$ in to expression is $4 \times 2 - 3$, that is $8 - 3 = 5$. Therefore when $x = 2$ is substituted in $4x - 3$ the value obtained is 5.

According to Sujith, he became the winner because he had learnt in grade 8 how to find the value of algebraic expressions by substituting integers.

6.2 Finding the value of expressions with one variable by substitution.

Example 1

Find the value of $4x-3$, when $x = -2$

$$\begin{aligned} 4x-3 &= 4 \times x - 3 \\ &= [4 \times (-2)] - 3 \\ &= (-8) - 3 \\ &= \underline{\underline{-11}} \end{aligned}$$

Example 2

Find the value of $4a-5$, when $a = \frac{1}{2}$

$$\begin{aligned} 4a-5 &= 4 \times a - 5 \\ &= 4 \times \frac{1}{2} - 5 \\ &= 2-5 \\ &= \underline{\underline{-3}} \end{aligned}$$

Example 3

Find the value of $5p+1$, when $p = -\frac{2}{3}$

$$\begin{aligned} 5p+1 &= 5 \times p + 1 \\ &= \left[\frac{5}{1} \times \left(-\frac{2}{3} \right) \right] + 1 \\ &= \left(-\frac{10}{3} \right) + \frac{1}{1} \\ &= -3\frac{1}{3} + 1 \\ &= \underline{\underline{-2\frac{1}{3}}} \end{aligned}$$



Exercise 6.1



1. Find the value of the following expressions when $x = \frac{2}{3}$

(i) $2x$ (ii) $3x$ (iii) $4x$ (iv) $5x$

2. Fill in the blanks.

(i) $3x + 1$, when $x = 2$
 $= 3 \times x + 1$
 $= 3 \times \text{---} + \text{---}$
 $= \text{---} + \text{---}$
 $= \text{---}$

(ii) $3a - 1$ when $a = \frac{1}{3}$
 $= 3 \times a - 1$
 $= \text{---} - 1$
 $= \text{---} - 1$
 $= \text{---}$

(iii) $2p + 3$ when $p = -\frac{3}{4}$
 $= 2 \times p + 3$
 $= \text{---} + \text{---}$
 $= \text{---} + \text{---}$
 $= \text{---}$

(3) Find the value of each of the following expressions when $x = 3$, $a = -2$, $p = \frac{1}{3}$, and $y = -\frac{2}{3}$

(i) $2x + 5$

(ii) $3a + 8$

(iii) $3p + 2$

(iv) $3y - 1$

(v) $5 - 3x$

(vi) $a - 7$

(vii) $2 + 2p$

(viii) $2y + 3$

(ix) $\frac{2}{3}x + 1$

(x) $10 + 2p$

(xi) $5 - 3p$

(xii) $-y + 2$

6.3 Finding by substitution the value of algebraic expressions with more than one variable

Example 4

Find the value of $2x - 3y$, if $x = 3$ and $y = \frac{1}{2}$

$$2x - 3y = 2 \times x - 3 \times y$$

$$= (2 \times 3) - \left(3 \times \frac{1}{2}\right)$$

$$= 6 - \frac{3}{2}$$

$$= \frac{12 - 3}{2}$$

$$= \frac{9}{2} = 4\frac{1}{2}$$

Example 5

Find the value of $2a - 3b$, if $a = 3$ and $b = -\frac{1}{2}$

$$2a - 3b = (2 \times a) - (3 \times b)$$

$$= (2 \times 3) - \left[3 \times \left(-\frac{1}{2}\right)\right]$$

$$= 6 + \frac{3}{2} = 6 + 1\frac{1}{2} = 7\frac{1}{2}$$

Example 6

Find the value of $2a+3b-c$, when $a = \frac{2}{5}$, $b = -\frac{1}{3}$, $c = 2$

$$\begin{aligned} 2a+3b-c &= (2 \times a) + (3 \times b) - c \\ &= \left(2 \times \frac{2}{5}\right) + \left[3^1 \times \left(-\frac{1}{3^1}\right)\right] - 2 \\ &= \frac{4}{5} - 1 - 2 = \frac{4}{5} - 3 = \frac{4}{5} - \frac{15}{5} = -\frac{11}{5} = -2\frac{1}{5} \end{aligned}$$

Exercise 6.2

- Find the value of the following expressions when $x = 2$ and $y = -3$
 - $2x + 3y$
 - $3x + 2y$
 - $5x - 3y$
 - $x - 5y$
- Find the value of the following expressions when $a = 3$ and $b = \frac{3}{4}$
 - $3a - 4b$
 - $2a + b$
 - $a - 3b$
 - $3a - 2b$
 - $a + 2b - 6$
 - $5a - 3b$
- Find the value of the following expressions when $p = \frac{1}{2}$, $q = -\frac{1}{3}$ and $r = 2$
 - $2p + 3q$
 - $4p + 3q + r$
 - $p + q + r$
 - $5p - 2q + 2r$
 - $p - 6q + 2r$
 - $3p - q - 2r$
- Complete the table given below along rows and find the value of 'y' corresponding to each value of 'x'.

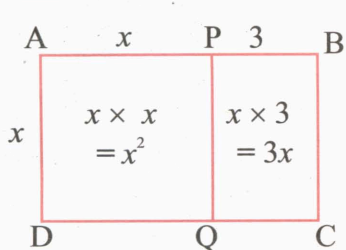
$$y = 2x + 3$$

x	-2	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
2x			-2				
+3	+3	+3	+3	+3	+3	+3	+3
2x+3			$-2+3=1$				
y			1				

- The circumference of a circle is $2\pi r$. Find the circumference of the circle if

$$\pi = \frac{22}{7} \text{ and } r = 3\frac{1}{2} \text{ cm}$$

6.4 Multiplication of a binomial expression by a binomial expression

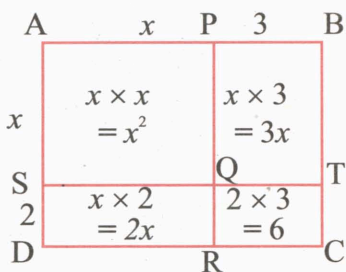


Expressions consisting of two terms such as an algebraic term and an algebraic term or a number connected with a + or - sign are called binomial expressions.

You have learnt in grade 8 how to simplify expressions such as $x(x+3)$ by removing brackets. The following shows how an algebraic expression is multiplied by an algebraic term

$$x(x+3) = x^2 + 3x$$

In the figure given below, the rectangle ABCD is divided into the square APQS and to the rectangles PBTQ, SQRD and QTCR



$$\text{The length at the rectangle ABCD} = (x+3)$$

$$\text{The breadth at the rectangle ABCD} = (x+2)$$

$$\text{The area at the rectangle ABCD} = (x+3)(x+2)$$

$$\left. \begin{array}{l} \text{When taken separately the area} \\ \text{of the rectangle ABCD} \end{array} \right\} = \begin{array}{l} \text{Area of APQS} + \text{Area of PBTQ} + \\ \text{Area of SQRD} + \text{Area of QTCR.} \\ = x^2 + 3x + 2x + 6 \end{array}$$

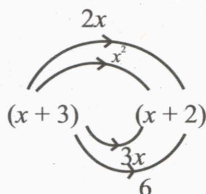
Since it is the same area of ABCD that is obtained in both occasions

$$(x+3)(x+2)$$

$$= x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$

Accordingly, the product of two binomial expressions can be obtained as follows.



$$(x+3)(x+2) = x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$

Example 7

$$\begin{aligned}
 \text{Simplify : } & (x+5)(x+2) \\
 & (x+5)(x+2) \\
 & = x(x+2) + 5(x+2) \\
 & = x^2 + 2x + 5x + 10 \\
 & = \underline{\underline{x^2 + 7x + 10}}
 \end{aligned}$$

Example 8

$$\begin{aligned}
 \text{Simplify : } & (x+a)(x+b) \\
 & (x+a)(x+b) \\
 & = x(x+b) + a(x+b) \\
 & = x^2 + bx + ax + ab \\
 & = \underline{\underline{x^2 + x(a+b) + ab}}
 \end{aligned}$$

Exercise 6.3

1. Simplify the following binomial expressions.

- | | | |
|--------------------|---------------------|--------------------|
| (i) $(x+5)(x+3)$ | (ii) $(x+1)(x+10)$ | (iii) $(3+x)(2+x)$ |
| (iv) $(a+5)(a+3)$ | (v) $(p+7)(p+a)$ | (vi) $(p+a)(p+b)$ |
| (vii) $(2+y)(3+y)$ | (viii) $(y+6)(y+1)$ | (ix) $(m+5)(m+2)$ |
| (x) $(10+a)(a+3)$ | | |

2. (i) Draw a sketch with the relevant measurements to show the area of a rectangle expressed by $(x+4)(x+2)$

(ii) Using the sketch show that the area of the rectangle is $x^2 + 2x + 4x + 8$

(iii) If the length of the rectangle of (i) above is increased by 2 units and the breadth is decreased by 1 unit; show that the area of the new rectangle is $x^2 + 7x + 6$

(iv) If $x = 5$, show that the area of the rectangle with the alteration of (iii) above is 3 square units greater than the area of the first rectangle.

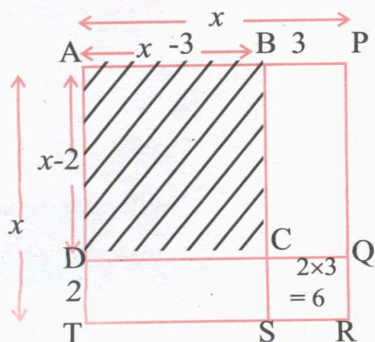
3. There is a road of breadth x metres around a square playground of which the length of one side is 30 m.

(i) Express the area of the play ground and the road in terms of x .

(ii) Express the area of the road in terms of x .

(iii) If $x = 5$ m find the area of the road.

6.5 Product of two binomial expressions (Further)



The length of the shaded rectangle ABCD = $x - 2$

The breadth of the shaded rectangle ABCD = $x - 3$

The area of the shaded rectangle ABCD

$$= (x - 2)(x - 3)$$

The area of the rectangle ABCD

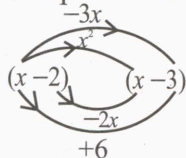
$$\begin{aligned} \text{when taken in parts} &= \text{Area of APRT} - \text{Area of BPRS} - \text{Area of DQRT} + \\ &\quad \text{Area of CQRS.} \\ &= x^2 - 3x - 2x + 6 \\ &= x^2 - 5x + 6 \end{aligned}$$

(As the area of CQRS was subtracted twice, it is added once at the end)

As it is the area of the shaded part ABCD that is obtained in both instances,

$$(x-2)(x-3) = x^2 - 3x - 2x + 6 = x^2 - 5x + 6$$

It is clear that the product of two binomial expressions can be obtained as follows.



$$\begin{aligned} &x^2 - 3x - 2x + 6 \\ &= x^2 - 5x + 6 \end{aligned}$$

$x \times x$	$= x^2$
$x \times (-3)$	$= -3x$
$x \times (-2)$	$= -2x$
$(-2) \times (-3)$	$= 6$

Example 9

$$\begin{aligned} \text{Simplify: } &(x-5)(x-1) \\ &(x-5)(x-1) \\ &= x(x-1) - 5(x-1) \\ &= x^2 - x - 5x + 5 \\ &= \underline{x^2 - 6x + 5} \end{aligned}$$

Example 10

$$\begin{aligned} \text{Simplify: } &(x-5)(x+2) \\ &(x-5)(x+2) \\ &= x^2 + 2x - 5x - 10 \\ &= \underline{x^2 - 3x - 10} \end{aligned}$$

Example 11

$$\begin{aligned} \text{Simplify: } &(x-5)(x+5) \\ &(x-5)(x+5) \\ &= x(x+5) - 5(x+5) \\ &= x^2 + 5x - 5x - 25 \\ &= \underline{x^2 - 25} \end{aligned}$$

Example 12

$$\begin{aligned} \text{Simplify: } &(x-a)(x-b) \\ &(x-a)(x-b) \\ &= x^2 - bx - ax + ab \\ &= \underline{x^2 - (a+b)x + ab} \end{aligned}$$

Exercise 6.4

1. Simplify the following products of binomial expressions.

(i) $(x-3)(x-7)$

(ii) $(x-1)(x-10)$

(iii) $(5-x)(2-x)$

(iv) $(x-7)(x+1)$

(v) $(a+2)(a-5)$

(vi) $(p-7)(p+3)$

(vii) $(a-10)(a-5)$

(viii) $(10-p)(2-p)$

(ix) $(a+3)(8-a)$

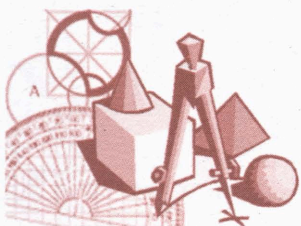
(x) $(7+a)(7-a)$

2. The length of a rectangle is x units and the breadth is y units. Its length is reduced by 2 units and the breadth by 1 unit. Express in terms of x and y ,

(i) the length,

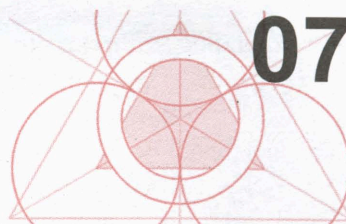
(ii) the breadth,

(iii) the area, of the new rectangle.



Factors

07



By studying this chapter you will be able to achieve the following competencies.

- ★ Expressing algebraic expressions with four terms as a product of two factors by grouping the terms in pairs and taking out the common factors.
- ★ Resolving quadratic expressions into factors correctly.
- ★ Resolving into factors a difference of two squares.

To recollect what you have already learnt in grade 8, find the factors of the following algebraic expressions.

Exercise 7.1

(i) $2k - 12$

(iii) $2ab - 8a + 4a^2$

(v) $30y^2 - 6y - 6$

(vii) $12a^3 - 36a^2b - 24ab^2$

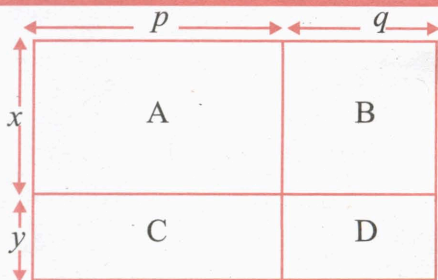
(ii) $3x^2 - 5xy$

(iv) $5x^2 - 15xy - 20xy^2$

(vi) $8c^2 - 6cd + 2c$

(viii) $6p - 24p^2 + 30p^3$

7.1 Factors of expressions with four terms.



The area of the rectangle shown in the figure is equal to the sum of the areas of the rectangles A, B, C, and D

Taking the parts together and finding the area.

The sum of the areas of parts A, B, C, D

The length of the whole figure

The breadth of the whole figure

The area of the whole figure

Therefore $px + qx + py + qy$

$$= px + qx + py + qy$$

$$= p + q$$

$$= x + y$$

$$= (p + q)(x + y)$$

$$= (p + q)(x + y)$$

∴ $(p + q)$ and $(x + y)$ are the factors of the expression $px + qx + py + qy$

By expanding $(p+q)(x+y)$ you will realise the truth of it

$$\begin{aligned}(p+q)(x+y) &= p(x+y) + q(x+y) \\ &= px + py + qx + qy\end{aligned}$$

The fact that,

$px + py + qx + qy = (p+q)(x+y)$ can be obtained by grouping the terms into pairs and taking out the common factor.

$$\begin{aligned}px + py + qx + qy &= p(x+y) + q(x+y) \\ &= \underline{(x+y)(p+q)}\end{aligned}$$

Example 1

Find the factors of $3a - 6c + 2ak - 4ck$. By grouping into pairs and taking the common factors out the expression can be written as,

$$\begin{aligned}3a - 6c + 2ak - 4ck &= 3(a - 2c) + 2k(a - 2c) \\ &= (a - 2c)(3 + 2k) \quad [(a - 2c) \text{ is a common factor}]\end{aligned}$$

The accuracy can be checked by expanding $(a - 2c)(3 + 2k)$

$$\begin{aligned}(a - 2c)(3 + 2k) &= a(3 + 2k) - 2c(3 + 2k) \\ &= 3a + 2ak - 6c - 4ck\end{aligned}$$

Example 2

Find the factors of: $x^2 + xy - x - y$

$$\begin{aligned}x^2 + xy - x - y &= x(x+y) - 1(x+y) \\ &= \underline{(x+y)(x-1)}\end{aligned}$$

Example 3

Find the factors of: $c^2 - 3c + bc - 3b$

$$\begin{aligned}c^2 - 3c + bc - 3b &= c(c-3) + b(c-3) \\ &= \underline{(c-3)(c+b)}\end{aligned}$$

Exercise 7.2

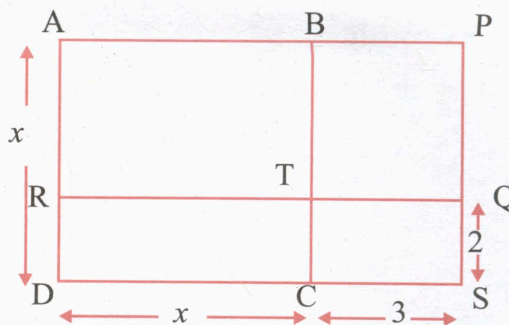
Find the factors of the following expressions by grouping into pairs and taking out the common factor. Check the accuracy by multiplying the factors.

- | | |
|---------------------------|---------------------------|
| 1. $ab + ac + 2b + 2c$ | 2. $p^2 - pq + 3pr - 3qr$ |
| 3. $ax - ay - bx + by$ | 4. $pr + pt - qr - qt$ |
| 5. $2pq + 6ps - 5q - 15s$ | 6. $x^2 + 2xy - 3x - 6y$ |
| 7. $2ab - 2ac + b - c$ | 8. $x^2 - 3xy - 6x + 18y$ |
| 9. $4 - 4a + c - ac$ | 10. $k - kl - l + l^2$ |

7.2 Factors of quadratic expressions.

The length of a side of the square ABCD shown in the figure is x units, while the length of PB is 3 units. Also the length of DR is 2 units. Let us find the area of the rectangle APQR.

The length of AP = $x+3$
 The breadth of AR = $x-2$
 The area of APQR = $(x+3)(x-2)$



The area of APQR = The area of APSD - the area of SDRQ
 $= x(x+3) - 2(x+3)$
 $= x^2 + 3x - 2x - 6$
 $= \underline{\underline{x^2 + x - 6}}$

This area could also be obtained in the following way.

The area of APQR = The area of ABTR + the area of BPQT
 $= x(x-2) + 3(x-2)$
 $= x^2 - 2x + 3x - 6$
 $= \underline{\underline{x^2 + x - 6}}$

When all these expressions are considered,

$$x^2 + x - 6 = (x+3)(x-2)$$

Accordingly the factors of $x^2 + x - 6$ are $(x+3)$ and $(x-2)$

In the algebraic expression $x^2 + x - 6$, the product of the term x^2 and the constant term gives $-6x^2$. The linear factors of $-6x^2$ are as follows.

$$\begin{array}{l}
 \nearrow \\
 \rightarrow \\
 \rightarrow \\
 \searrow
 \end{array}
 \begin{array}{l}
 (-6x) \times (+x) \\
 (+6x) \times (-x) \\
 \boxed{(+3x) \times (-2x)} \\
 (-3x) \times (+2x)
 \end{array}$$

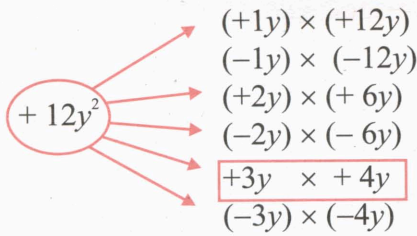
For the algebraic sum of the pair of factors to be $+x$, which is the middle term of the expression, the pair of factors that should be selected is $+3x$ and $-2x$.

Then the expression $x^2 + x - 6$ can be written as $x^2 + 3x - 2x - 6$

$$\begin{aligned}
 x^2 + x - 6 &= x^2 + 3x - 2x - 6 \\
 &= x(x+3) - 2(x+3) \\
 &= \underline{\underline{(x+3)(x-2)}}
 \end{aligned}$$

Example 4

Find the factors of $y^2 + 7y + 12$



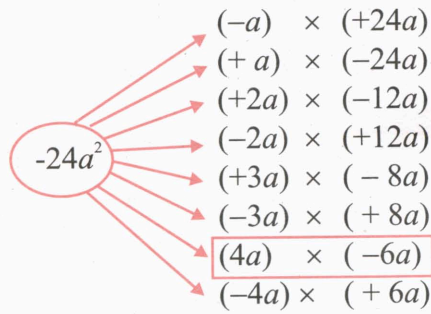
The product of the y^2 term and the constant term is $+12y^2$
 The pair of factors that has the middle term of the expression $+7y$ as its sum is $+3y$ and $+4y$ only.

Therefore

$$\begin{aligned} y^2 + 7y + 12 \\ &= y^2 + 3y + 4y + 12 \\ &= y(y+3) + 4(y+3) \\ &= \underline{\underline{(y+3)(y+4)}} \end{aligned}$$

Example 5

Find the factors of $a^2 - 2a - 24$



The product of the term a^2 and the constant term is $-24a^2$

Out of these pairs of factors the suitable pair which leads to the middle term $-2a$ is $+4a$ and $-6a$

$$\begin{aligned} a^2 - 2a - 24 \\ &= a^2 + 4a - 6a - 24 \quad (\text{the expression should be formed as shown here}) \\ &= a(a+4) - 6(a+4) \\ &= \underline{\underline{(a+4)(a-6)}} \end{aligned}$$

Example 6

Find the factors of: $30 - 17k + k^2$

The product of the constant term and the k^2 term is $+30k^2$

Out of the pairs of factors of $+30k^2$ the suitable pair which leads to the middle term $-17k$ is the pair of factors $-2k$ and $-15k$.

Accordingly

$$\begin{aligned} 30 - 17k + k^2 \\ &= 30 - 2k - 15k + k^2 \quad (\text{the expression should be formed as shown here}) \\ &= 2(15 - k) - k(15 - k) \quad (2 \text{ and } -k \text{ are common factors}) \\ &= \underline{\underline{(15 - k)(2 - k)}} \end{aligned}$$

When a quadratic expression is given with a common factor, the common factor should be taken out first. Then the quadratic expression within brackets should be resolved into factors.

Example 7

Find the factors of: $18 + 15a - 3a^2$

$$\begin{aligned} &18 + 15a - 3a^2 \\ &= 3(6 + 5a - a^2) \end{aligned}$$

The product of the a^2 term and the constant term is $-6a^2$.

Out of the factors of $-6a^2$, the correct pair which leads to the middle term $+5a$ of the expression is $-a$ and $+6a$

Therefore

$$\begin{aligned} &18 + 15a - 3a^2 \\ &= 3[6 + 5a - a^2] \\ &= 3[6 + 6a - a - a^2] \\ &= 3[6(1 + a) - a(1 + a)] \\ &= 3[(1 + a)(6 - a)] \\ &= \underline{\underline{3(1 + a)(6 - a)}} \end{aligned}$$



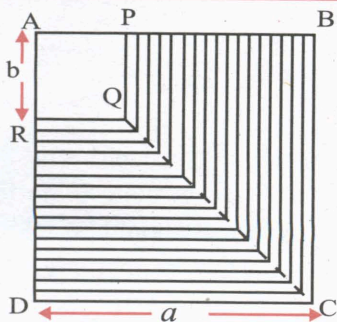
Exercise 7.3



Find the factors of the following quadratic expressions. Check the accuracy of the factors by writing the product of the factors.

- | | | |
|----------------------|-----------------------|-----------------------|
| 1. $a^2 + 8a + 12$ | 2. $y^2 + 3y - 18$ | 3. $p^2 - 3p - 40$ |
| 4. $q^2 - 11q + 24$ | 5. $r^2 - r - 30$ | 6. $l^2 - 19l + 18$ |
| 7. $s^2 + 3s - 70$ | 8. $c^2 + 9c + 20$ | 9. $36 + 15k + k^2$ |
| 10. $16 + 6x - x^2$ | 11. $30 - 7c - c^2$ | 12. $45 - 18y + y^2$ |
| 13. $24 + 23x - x^2$ | 14. $42 - 11z - z^2$ | 15. $54 + 15d - d^2$ |
| 16. $54 - 15f + f^2$ | 17. $3x^2 - 24x + 36$ | 18. $45 + 30y + 5y^2$ |
| 19. $72 - z - z^2$ | 20. $48 - 14g + g^2$ | |

7.4 Factors of a difference of two squares.



The given diagram shows how a square APQR, the length of a side of which is 'b', is placed inside another square ABCD, the length of a side of which is 'a'. Let us calculate the area of the shaded portion.

When the difference of the areas of the squares is considered the area of the shaded portion $= a^2 - b^2$
When the shaded portion is separated into two parts

The algebraic sum of the factors is zero only with the factors $+2y$ and $-2y$.

Therefore

$$\begin{aligned} & 3[1 - 4y^2] \\ &= 3[1 + 2y - 2y - 4y^2] \\ &= 3[1(1 + 2y) - 2y(1 + 2y)] \\ &= 3[(1 + 2y)(1 - 2y)] \\ &= \underline{\underline{3(1 + 2y)(1 - 2y)}} \end{aligned}$$



Exercise 7.4



Find the factors of the following expressions.

- | | | |
|------------------|-------------------|----------------------|
| 1. $y^2 - 9$ | 2. $p^2 - 36$ | 3. $25 - a^2$ |
| 4. $4 - 9k^2$ | 5. $4x^2 - 36y^2$ | 6. $a^2b^2 - 1$ |
| 7. $18c^2 - 2$ | 8. $4z^2 - 100$ | 9. $125k^2 - 5$ |
| 10. $27d^2 - 48$ | 11. $3x^3 - 243$ | 12. $5m^2 - 3125n^2$ |

When finding factors; changing the position of the terms of an expression should be done such that its accuracy is preserved.

Example 12

In the expression, $ax + by - ay - bx$, there are no factors common to the first two terms and no factors common to the last two terms.

When rewritten by interchanging the second, third and fourth terms or writing the 1st term at the end it is

$$\begin{aligned} & ax - ay - bx + by \\ &= a(x - y) - b(x - y) \\ &= \underline{\underline{(x - y)(a - b)}} \end{aligned}$$

$$\begin{aligned} & by - ay - bx + ax \\ &= y(b - a) - x(b - a) \\ &= \underline{\underline{(b - a)(y - x)}} \end{aligned}$$

Example 13

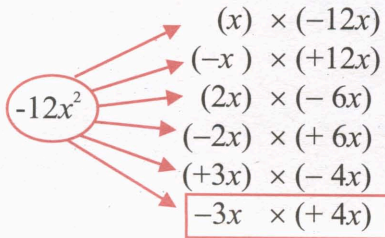
In the expression $pq - 6 + 3q - 2p$ there are no factors common to either the first two terms or the last two terms. When rewritten with the third term in the second place and the second term in the fourth place, the expression can be resolved into factors

$$\begin{aligned} & pq + 3q - 2p - 6 \\ &= q(p + 3) - 2(p + 3) \\ &= \underline{\underline{(p + 3)(q - 2)}} \end{aligned}$$

Example 14

Find the factors of: $x - 12 + x^2$

When rewritten with the third term in the first place it will be $x^2 + x - 12$



The pair of factors which leads to the middle term $+x$ are $-3x$ and $+4x$

$$\begin{aligned}
 &x^2 + x - 12 \\
 &= x^2 + 4x - 3x - 12 \\
 &= x(x+4) - 3(x+4) \\
 &= \underline{\underline{(x+4)(x-3)}}
 \end{aligned}$$

Example 15

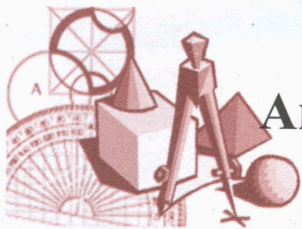
Find the factors of: $-4(3y-5)+y^2$

$$\begin{aligned}
 &-12y + 20 + y^2 \\
 &= y^2 - 12y + 20 \\
 &= y^2 - 10y - 2y + 20 \\
 &= y(y-10) - 2(y-10) \\
 &= \underline{\underline{(y-10)(y-2)}}
 \end{aligned}$$

Exercise 7.5

Find the factors of the following algebraic expressions.

- | | |
|-------------------------|------------------------|
| 1. $px^2 - 1 - x^2 + p$ | 2. $4 - k^2 - 3k$ |
| 3. $ax - by + ay - bx$ | 4. $3y - 28 + y^2$ |
| 5. $x^3 + 2 + 2x^2 + x$ | 6. $x^3 + 1 + x^2 + x$ |



Angles Associated With Straight Lines and Parallel Lines

By studying this chapter you will be able to achieve the competencies in relation to,

- ★ angles associated with straight lines.
- ★ the proof of the theorem on vertically opposite angles and its applications.
- ★ theorems associated with parallel lines and their applications.

8.1 Axioms and Theorems

You have learnt in earlier grades about different types of angles. The objective of this lesson is to further study about them. Let us first study a few axioms which are important in this process.

Axiom 1

If the same quantity is added to two equal quantities the results are equal.

$$\begin{array}{l} \text{That is, if} \quad a = b \\ \text{then} \quad a + c = b + c \end{array}$$

Axiom 2

If the same quantity is subtracted from two equal quantities the results are equal.

$$\begin{array}{l} \text{That is, if} \quad a = b \\ \text{then} \quad a - c = b - c \end{array}$$

Axiom 3

If two quantities are multiplied by the same quantity the results are equal.

$$\begin{array}{l} \text{That is, if} \quad a = b \\ \text{then} \quad na = nb \end{array}$$

Axiom 4

If two quantities are divided by the same non zero quantity the results are equal.

$$\begin{array}{l} \text{That is, if} \quad a = b \\ \text{then} \quad \frac{a}{n} = \frac{b}{n} \end{array}$$

Axiom 5

Quantities equal to the same quantity are equal.

$$\begin{array}{l} \text{That is, if} \quad a = b \text{ and } a = c \\ \text{then} \quad b = c \end{array}$$

These axioms can be used in geometrical proofs.

Example 1

The diagram shows that
Show that

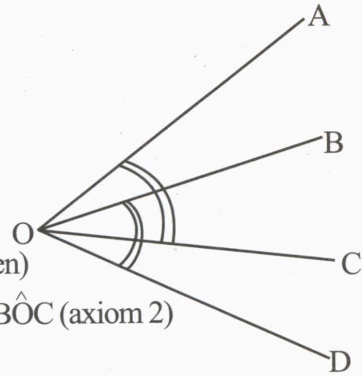
$$\begin{aligned} AB &= CD \\ AC &= BD \\ AB &= CD \\ AB + BC &= BC + CD \text{ (axiom 1)} \\ AC &= BD \end{aligned}$$



Example 2

The diagram shows that
Show that

$$\begin{aligned} \hat{AOC} &= \hat{BOD} \\ \hat{AOB} &= \hat{COD} \\ \hat{AOC} &= \hat{BOD} \text{ (given)} \\ \hat{AOC} - \hat{BOC} &= \hat{BOD} - \hat{BOC} \text{ (axiom 2)} \\ \underline{\hat{AOB}} &= \underline{\hat{COD}} \end{aligned}$$



Exercise 8.1

1. Using axioms and the relations given below, write conclusions that can be arrived at.

(i) $PQ = RS$
 $PQ = ST$

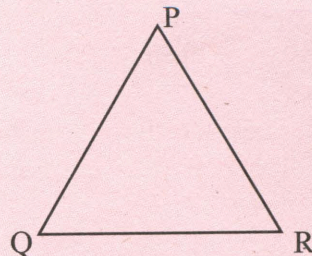
(ii) $x + y = 180^\circ$
 $p + q = 180^\circ$

(iii) $\hat{POQ} = 30^\circ$
 $\hat{RST} = 30^\circ$

(iv) $LM = 3.5 \text{ cm}$
 $MN = 3.5 \text{ cm}$

2. Using the diagrams given below, write conclusions that can be arrived at.

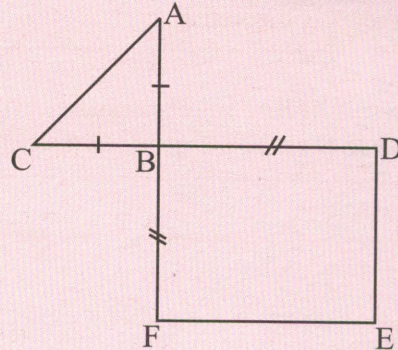
(i) In triangle PQR
 $PQ = PR$
 $PR = QR$



(ii) In the given diagram

$$AB = BC$$

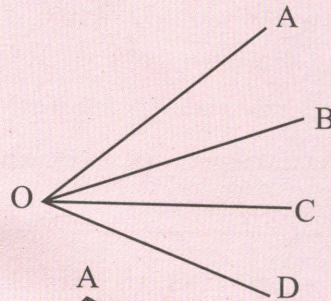
$$BD = BF$$



(iii) In the given diagram

$$\hat{A}OB = \hat{B}OC$$

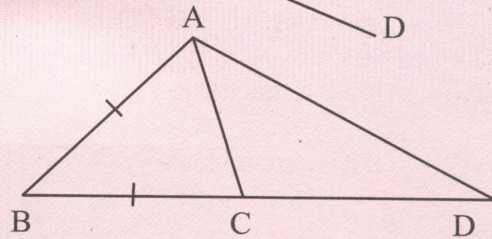
$$\hat{A}OC = \hat{B}OD$$



(iv) In triangle ABD

C is the mid point of BD

$$BC = BA$$



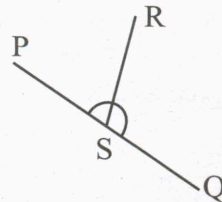
8.2 Adjacent angles and vertically opposite angles

In the given diagram the straight line RS meets the straight line PQ. You have learnt earlier that the angles PSR and RSQ so formed are supplementary adjacent angles.

That is,

$$\hat{P}SR + \hat{R}SQ = 180^\circ$$

This can be written as a theorem.



The mathematician Euclid who lived in 300 BC has compiled the book 'Elements' which includes a large number of theorems in an order that can be used in Geometry. At present we follow that order of theorems in geometry. Statements that can be proved logically with reasons using axioms are known as theorems.

As this is considered as an elementary theorem; this will be used with axioms, to prove the other theorems.

Theorem

If a straight line meets another straight line, the sum of the two adjacent angles so formed is equal to two right angles.

Activity 1

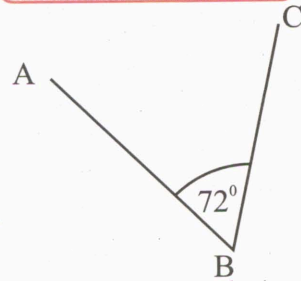


Draw angle $\hat{A}BC = 72^\circ$ as shown in the diagram. Calculate the magnitude of the supplementary angle of $\hat{A}BC$

This will be $180^\circ - 72^\circ = 108^\circ$.

Draw an angle of 108° such that CB is a common arm and, B is the vertex. Name it as $\hat{C}BD$

Using a straight edge check whether ABD is a straight line. What is the conclusion that you can arrive at?



The straight lines AB and PQ intersect each other. The magnitudes of the angles formed are shown as a, b, c and d .

Now, as AB is a straight line,

$$a + b = 180^\circ \text{--- (1) (Theorem above)}$$

Also, as PQ is a straight line

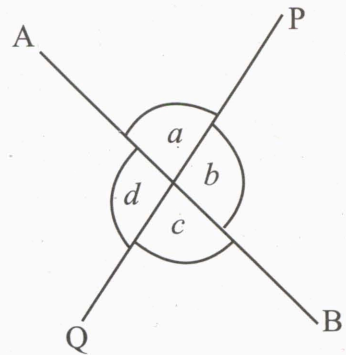
$$b + c = 180^\circ \text{--- (2)}$$

From (1) and (2)

$$a + b = b + c \text{ (Axiom 5)}$$

$$a + b - b = b + c - b \text{ (Axiom 2)}$$

$$\therefore a = c$$



Now it is clear to you that the vertically opposite angles formed when two straight lines intersect each other can be shown to be equal logically giving reasons.

This process is called proof of the theorem

A Proof

A proof is a process of arriving at conclusions based on logical reasoning using axioms and theorems proved earlier.

Let us prove the above theorem formally.

Theorem

If two straight lines intersect, the vertically opposite angles so formed are equal.

Data : The straight lines AB and CD intersect at O

To prove that : $\hat{AOC} = \hat{DOB}$ and $\hat{AOD} = \hat{COB}$

Proof :

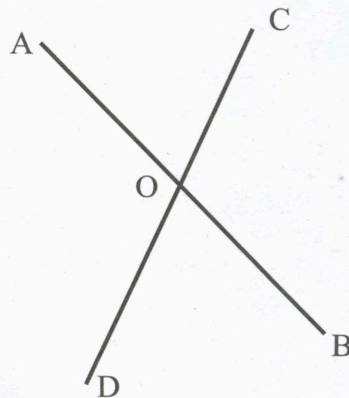
$$\hat{AOC} + \hat{COB} = 180^\circ \text{ -- (1) (AB is a straight line)}$$

$$\hat{COB} + \hat{BOD} = 180^\circ \text{ ----- (2) (CD is a straight line)}$$

From (1) and (2)

$$\hat{AOC} + \hat{COB} = \hat{COB} + \hat{BOD} \text{ (Axiom 5)}$$

$$\hat{AOC} = \hat{BOD} \text{ (axiom 2)}$$



Similarly,

$$\hat{COB} + \hat{BOD} = 180^\circ \text{ ----- (3) (CD is a straight line)}$$

$$\hat{AOD} + \hat{BOD} = 180^\circ \text{ ----- (4) (AB is a straight line)}$$

From (2) and (3)

$$\hat{COB} + \hat{BOD} = \hat{AOD} + \hat{BOD} \text{ (Axiom 5)}$$

$$\therefore \hat{COB} = \hat{AOD} \text{ (Axiom 2)}$$

Using the two theorems learnt so far, problems such as those given below can be solved.

Example 3

The diagram shows straight lines PQ, RS and ST.

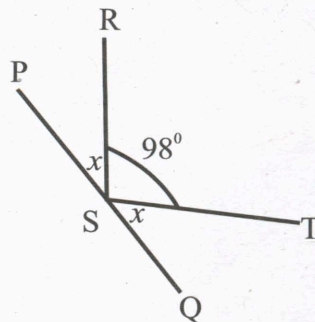
Find the value of x

$$x^\circ + 98^\circ + x^\circ = 180^\circ \text{ (PQ is a straight line)}$$

$$2x = 180^\circ - 98^\circ$$

$$2x = 82^\circ$$

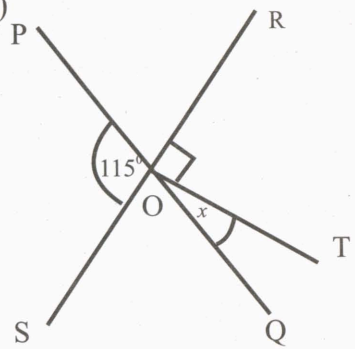
$$\therefore \underline{\underline{x = 41^\circ}}$$



Example 4

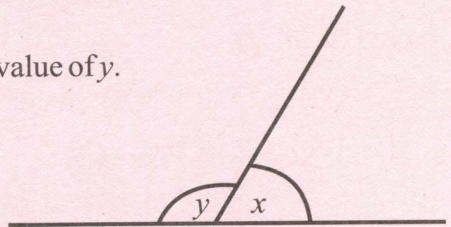
The straight lines PQ and RS shown in the diagram intersect at O. Find the value of x .

$$\begin{aligned} \hat{P}OS &= \hat{R}OQ \text{ (vertically opposite angles)} \\ \hat{P}OS &= 115^\circ \\ \text{But } \hat{R}OQ &= \hat{R}OT + \hat{T}OQ \\ 115^\circ &= 90^\circ + x^\circ \\ 90^\circ + x^\circ &= 115^\circ \\ \underline{\underline{x^\circ}} &= \underline{\underline{25^\circ}} \end{aligned}$$

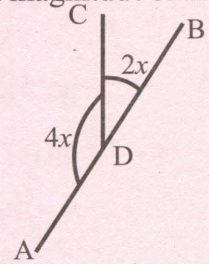


Exercise 8.2

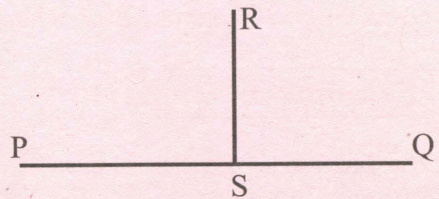
1. In the given diagram, if $x = 75^\circ$, find the value of y .



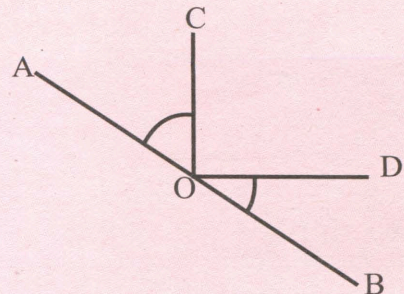
2. In the given diagram AB and CD are straight lines. Find the magnitude of the angles $\hat{B}DC$, and $\hat{A}DC$



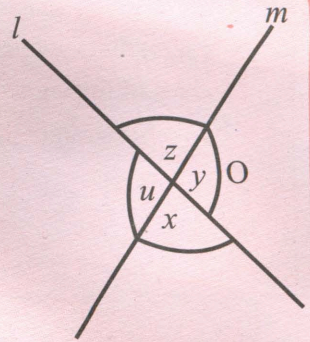
3. In the given diagram PQ and RS are straight lines $\hat{P}SR = \hat{R}SQ$. Find the magnitude of $\hat{P}SR$



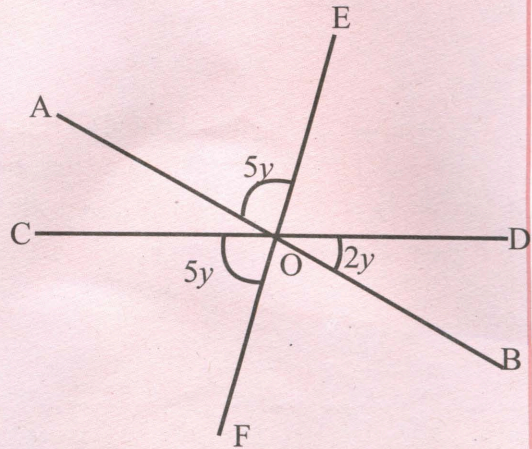
4. In the given diagram AB, CO, OD, are straight lines. $\hat{A}OC + \hat{B}OD = 90^\circ$
Find the magnitude of $\hat{C}OD$.



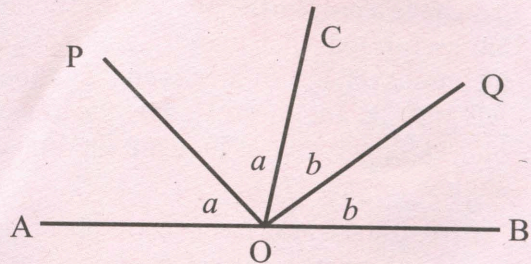
5. In the given diagram, lines l and m intersect at O . If $x = 45^\circ$, find the value of y , z , and u .



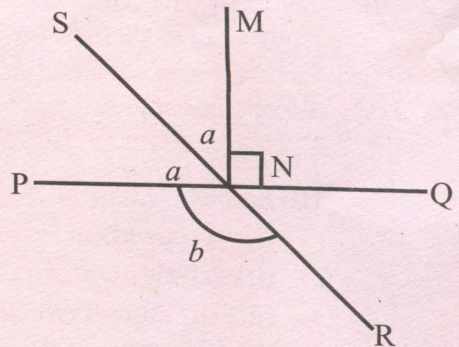
6. The straight lines AB , CD , and EF of the given diagram intersect at O . Find the value of y .



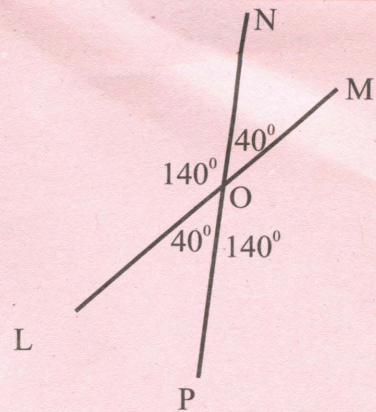
7. In the given diagram OP bisects \hat{AOC} and OQ bisects \hat{COB} . Show that $\hat{POQ} = 90^\circ$



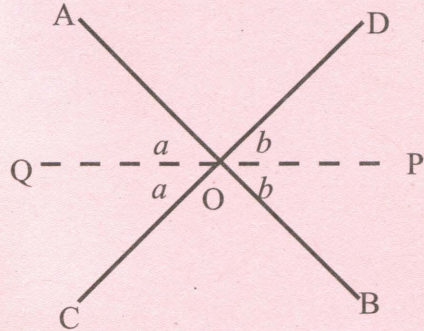
8. In the given diagram PQ , SR and MN are straight lines. Find the value of ' a ' and ' b '.



9. In the given diagram NO, LO, PO, MO, are straight lines which meet at O. Observe the values of the angles and giving reasons, name two other straight lines.



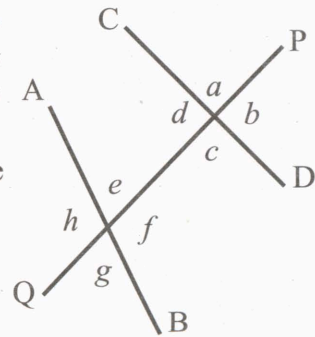
10. In the given diagram AB and CD are straight lines. OP and OQ are the bisectors of $\hat{D}OB$ and $\hat{A}OC$ respectively. Give reasons for QOP to be a straight line.



8.3 Angles associated with parallel lines.

You have learnt earlier about corresponding angles, alternate angles and allied angles formed when two straight lines are intersected by a transversal. In the given diagram the two lines AB and CD are intersected by the transversal PQ. a, b, c, d, e, f, g and h , indicate the magnitude of the angles.

- b and f are a pair of corresponding angles. Name three other pairs of corresponding angles.
- c and e are a pair of alternate angles. Name another pair of alternate angles.
- c and f are a pair of allied angles. Name another pair of allied angles.



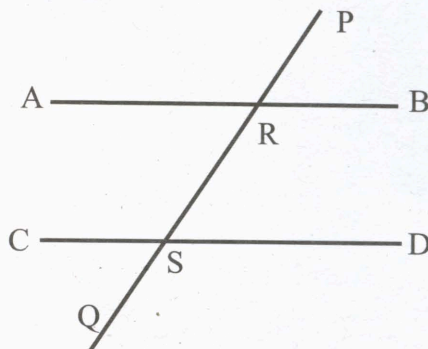
Theorem

If a transversal intersects two straight lines so as to make,

- a pair of corresponding angles equal or
 - a pair of alternate angles equal or
 - the sum of two allied angles equal to 180°
- the two straight lines are parallel.

This theorem will also be considered as an elementary theorem and its applications will be done without proving it formally. The diagram shows the transversal PQ intersecting the lines AB and CD.

- (i) If a pair of corresponding angles out of $\hat{P}RB$ and $\hat{R}SD$, $\hat{B}RS$ and $\hat{D}SQ$, $\hat{A}RP$ and $\hat{C}SR$, $\hat{A}RS$ and $\hat{C}SQ$ is equal then AB and CD are parallel.



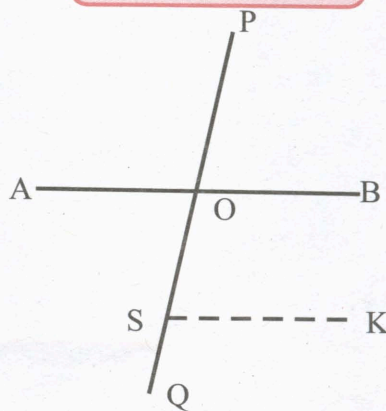
- (ii) If a pair of alternate angles out of $\hat{B}RS$ and $\hat{C}SR$ and $\hat{A}RS$ and $\hat{R}SD$ is equal, then AB and CD are parallel.

- (iii) If the sum of two allied angles out of $\hat{B}RS$ and $\hat{R}SD$, $\hat{A}RS$ and $\hat{R}SC$ is equal to 180° , then AB and CD are parallel.



1. Draw the two intersecting lines AB and PQ and name the point of intersection as O.
2. Measure angle $\hat{P}OB$ with a protractor
3. Mark a point S on OQ, and a point K lying on the same side as B such that $\hat{P}OB = \hat{O}SK$, and joined SK.
4. Use set squares and check whether AB and SK are parallel. What is your conclusion?

Activity 2



Theorem

If a transversal intersects two parallel straight lines,

- (i) the corresponding angles formed are equal.
- (ii) the alternate angles formed are equal.
- (iii) the sum of two allied angles formed is equal to 180°

This is the converse of the previous theorem

The two parallel lines LM and NP are intersected by the line RS. The arrows pointing to the same direction indicate that the lines are parallel.

- (i) The corresponding angles are equal.

$$a = e$$

$$b = f$$

$$c = g$$

$$d = h$$

- (ii) The alternate angles are equal

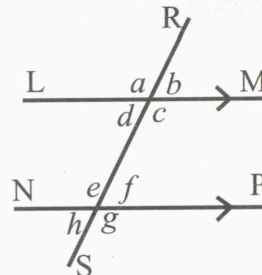
$$c = e$$

$$d = f$$

- (iii) The sum of two allied angles is 180°

$$c + f = 180^\circ$$

$$d + e = 180^\circ$$



Example 5

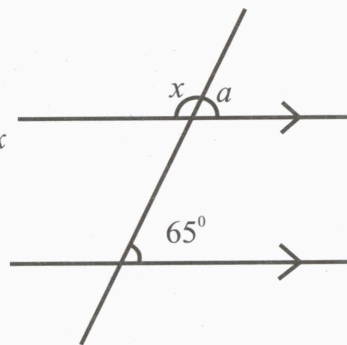
Using the data in the given diagram find the value of x

$$a = 65^\circ \text{ (Corresponding angles)}$$

$$x + a = 180^\circ \text{ (Adjacent angles on a straight line)}$$

$$\therefore x + 65^\circ = 180^\circ$$

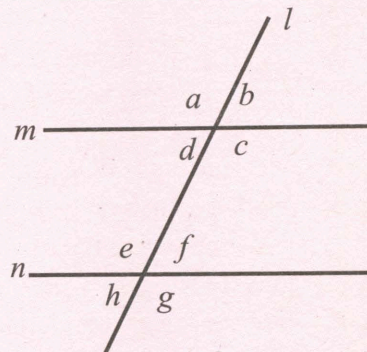
$$\therefore x = \underline{\underline{115^\circ}}$$



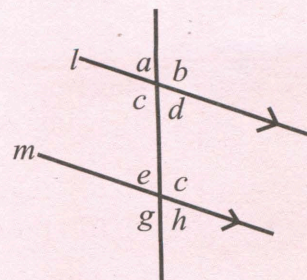
Exercise 8.3

1. In the given diagram l, m, n are straight lines.

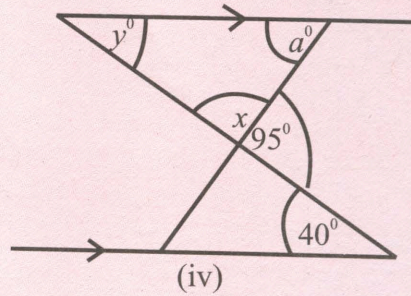
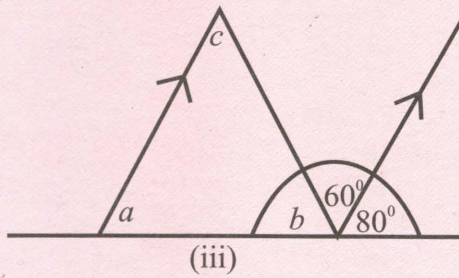
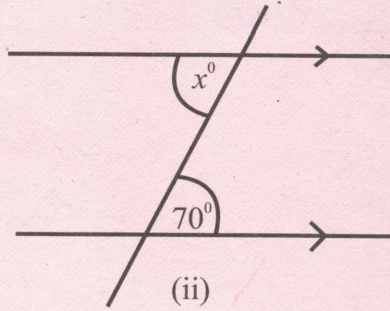
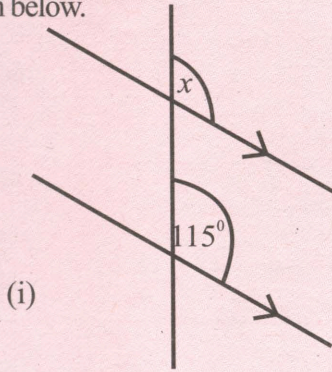
a, b, c, d, e, f, g, h indicate angles. If $a = 120^\circ$ and $f = 60^\circ$, show by giving reasons, that the lines m and n are parallel.



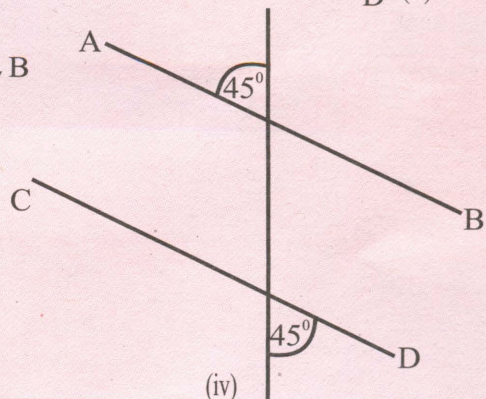
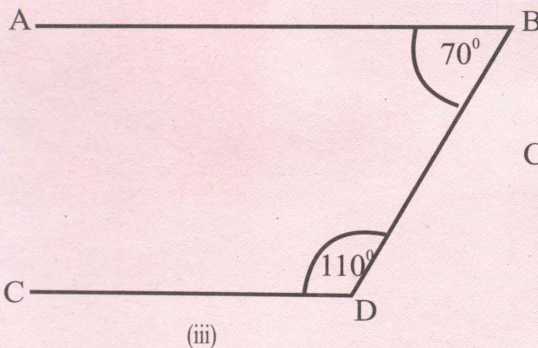
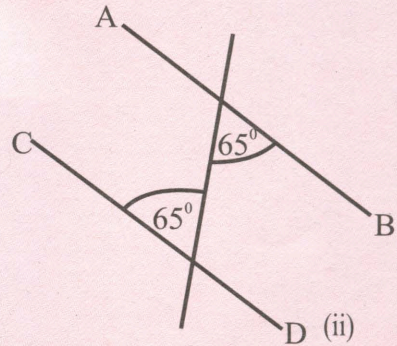
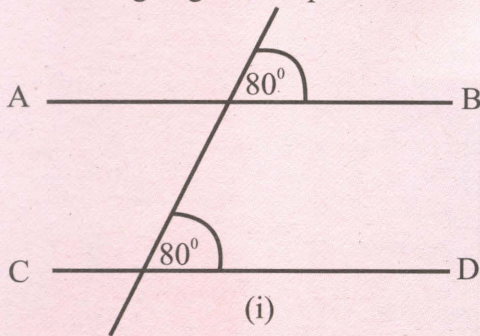
2. l and m are parallel lines in the given diagram. If $a = 47^\circ$, find the magnitude of all the other angles.



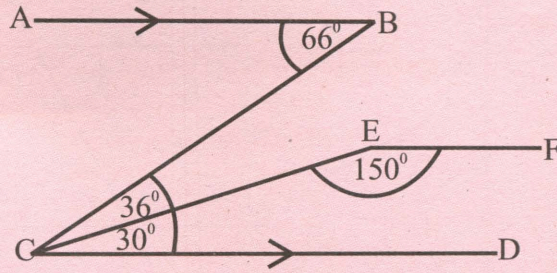
3. Find the magnitude of the angles indicated by algebraic symbols in the diagrams given below.

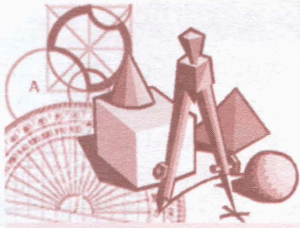


4. Express giving reasons whether the straight lines AB and CD in each of the following diagrams are parallel



5. Using the information given in the diagram show that AB is parallel to EF.





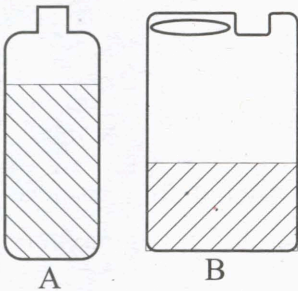
Liquid Measures

By studying this chapter you will be able to achieve the following competencies.

- ★ Developing the relationship between millilitres and cubic centimetres.
- ★ Developing the relationship between litres and cubic centimetres.
- ★ Developing the relationship between litres and cubic metres.
- ★ Solving problems using the above relationships.

You have learnt in grade 8 that the amount of space occupied by an object is called its volume. Also that the volume of the amount of liquid required to fill a vessel completely is called the capacity of that vessel.

9.1 Measures used in measuring volume and capacity.



The two containers A and B shown in the diagram contain a soft drink.

What quantities of soft drink are contained in them?

Which container has the greater quantity of the drink?

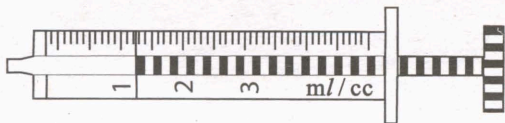
Answers to the above questions can be found as follows.

- (i) The quantity of the drink in each container can be measured using a cup.
- (ii) The drink in the two containers can be transferred into two identical bottles and the levels can be compared.
- (iii) A graduated measuring cylinder can be used and the quantities of drink can be compared.

In stage -

- (i) there can be a situation where the quantity of drink in the final cup is not measurable.
- (ii) the quantity of drink cannot be measured; it shows only which bottle has more drink.
- (iii) a graduated measuring cylinder is used. As such the exact volume can be read.

9.2 Relationship between millilitres and cubic centimetres.



Udara, a member of the "New Inventor Society" brought to the class a syringe (which doctors use without its needle)

Lasith who observed it went to the teacher to get certain aspects clarified.

"Sir, I do not know what this m/ cc is written for. It may be that 'm/' stands for millilitres. I am not quite clear about what this 'cc' stands for."

"Yes son you are correct. ml stands for millilitres. Drugs that are injected to a patient's body are measured in millilitres. cc are the two first letters of the words cubic centimetres. In mathematics we express it as cm^3 . This syringe can hold 3ml or 3cm^3 of the drug," explained the teacher.

"Sir, here, there are two types of units. Why is that?"

"Only the volume of liquids is measured in mililitres. But liquid measures as well as the volume of solids can be expressed in the unit cm^3 , These units are seen on the measuring cylinders in the laboratory"

A volume of 1 ml of liquid is 1cm^3
 $1\text{ ml} = 1\text{ cm}^3$

Example 1

The internal dimensions of a container the shape of a cuboid are as follows. Length 7 cm, breadth 5 cm, height 4 cm. Express in mililitres, the maximum volume of water which the container can hold.

$$\begin{aligned}\text{The internal volume of the container} &= 7\text{ cm} \times 5\text{ cm} \times 4\text{ cm} \\ &= 140\text{ cm}^3\end{aligned}$$

$$\text{The maximum volume of water the container can hold} = \underline{140\text{ ml}}$$

$$(\text{as } 1\text{ ml}) = 1\text{ cm}^3$$

Example 2

A small barrel contains 2.5 l of oil. If 500 ml of oil can be filled into one bottle, Find the number of bottles needed to fill this 2.5 l of oil.

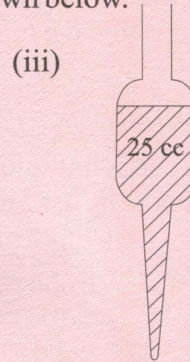
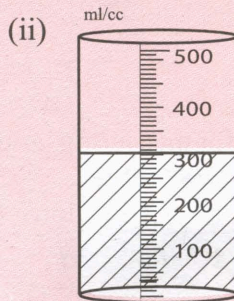
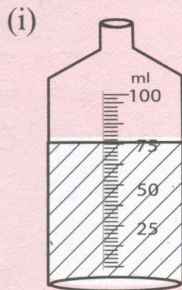
$$2.5\text{ l} = 2500\text{ ml}$$

$$\therefore \text{the number of bottles needed} = \frac{2500}{500}$$

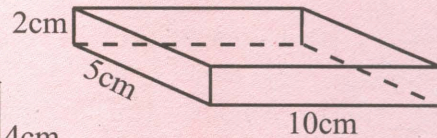
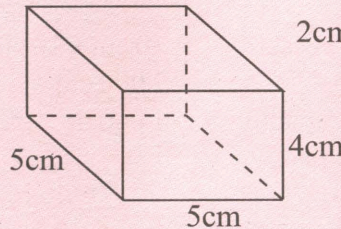
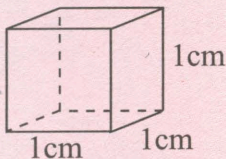
$$= \underline{5}$$

Exercise 9.1

1. Express in
 (i) millilitres (ii) cubic centimetres
 the volume of liquid found in the containers shown below.



2. Express in
 (i) cubic centimetres (ii) millilitres
 the capacity of the following containers.



3. Express in litres the volumes of liquid given below.
 (i) 2 000 ml (ii) 5 500 ml (iii) 1 200 ml (iv) 18 000 ml
 (v) 850 ml (vi) 200 ml (vii) 50 ml (viii) 300 ml
4. Express in millilitres the volumes of liquid given below.
 (i) 2 l (ii) 1.5 l (iii) 0.5 l (iv) 200 cm³
 (v) 50 cm³ (vi) 10 cc (vii) 200 cc (viii) 300 l
5. There is water to a height of 2 cm in a container the shape of a cube, one side of which is 10 cm. Express in millilitres the volume of water in the container.
6. A container the shape of a cuboid, 20 cm long 15 cm wide and 5 cm high, is filled with a liquid medicine. Into how many small bottles each of capacity 100 ml can this be refilled?
7. 1.5 l of a drink is to be served in equal amounts to 20 persons. Give in ml the quantity that each person will get.

8. 60 persons are to be served with a drink such that each person gets 100 ml. How many bottles each of 1.5 l of the drink should be bought for this purpose?
9. 50 persons are to be served with tea at a reception. There should be 180 ml of tea in each cup of tea. Assuming that 1 l of water is wasted as steam, and due to other reasons find the minimum volume of water that should be boiled.

9.3 The relationship between a litre and a cubic centimetre.

The unit ml or cm^3 is not suitable for measuring large quantities of liquid. A unit of a larger size should be used in such a situation.

As 1 000 ml is 1 l, 1 000 cm^3 too is 1 l,

$$1000 \text{ cm}^3 = 1000 \text{ ml} = 1 \text{ l}$$

Example 3

Express in (i) cm^3 (ii) ml (iii) l, the capacity of a vessel of the shape of a cube, a side of which is 10 cm

- (i) The capacity of the vessel $= 10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$
 $= 1000 \text{ cm}^3$
- (ii) As $1 \text{ cm}^3 = 1 \text{ ml}$ the capacity $= 1000 \text{ ml}$
- (iii) $1000 \text{ ml} = 1 \text{ l}$ the capacity $= 1 \text{ l}$

Example 4

If 2 l of water is poured into a container with a rectangular base of length 25 cm and breadth 20 cm, to what height will the water level rise?

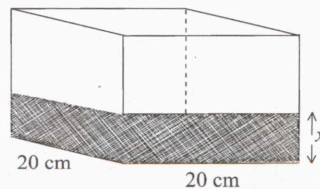
The volume of water in the container $= 25 \text{ cm} \times 20 \text{ cm} \times x$
 $= 2 \text{ l}$
 $= 2000 \text{ ml}$
 $= 2000 \text{ cm}^3$
 $= 2000$
 $= 2000$
 $= 4$

$$\therefore 25 \times 20 \times x = 2000$$

$$500x = 2000$$

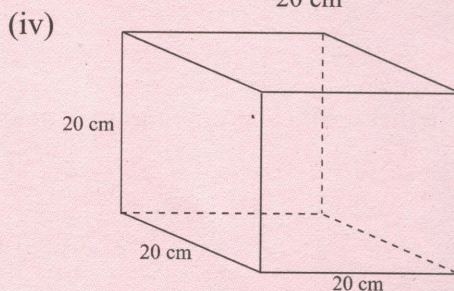
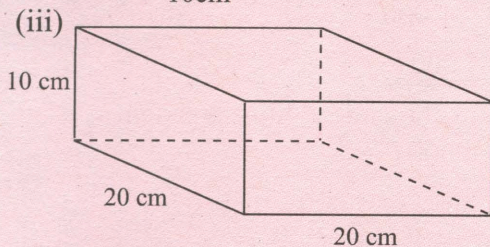
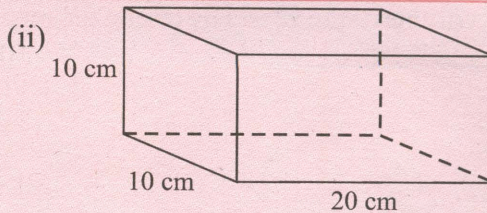
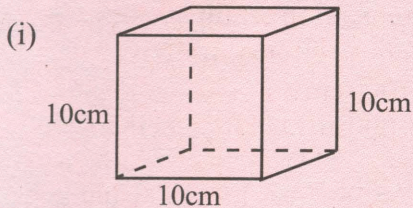
$$x = 4$$

\therefore The height of the water level will be 4 cm



Exercise 9.2

1. Express in (i) cubic centimetres (ii) millilitres (iii) litres the capacity (Maximum volume of water contained) of the following vessels.



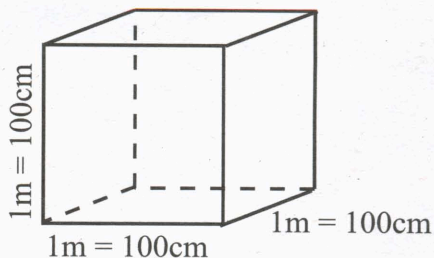
2. The area of the base of a tank the shape of a cuboid is 240 cm^2 . There is water in it, to a height of 40 cm. Find (i) in cm^3 (ii) in l , the volume of water in the tank.
3. Fill in the blanks of the table given below, assuming that the measurements are of cubical containers.

	Area of the base	The height of the level of water (cm)	Volume of water (cm^3)	Volume of water in (l)
(i)	2000 cm^2	30	-----	-----
(ii)	1000 cm^2	50	-----	-----
(iii)	400 cm^2	-----	-----	8
(iv)	500 cm^2	-----	-----	$2\frac{1}{2}$
(v)	$70 \text{ cm} \times 50 \text{ cm}$	-----	21 000	-----

9.4 The relationship between a Litre and a Cubic Metre

When calculating the volume of water contained in places like reservoirs, swimming pools, water tanks etc, the relevant unit of measurement should be looked into.

As $1 \text{ m} = 100 \text{ cm}$, let us find out how to obtain the volume of liquid, the vessel given below can hold.



The volume of the vessel by taking the measurements in metres
 $= 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$

The volume of the vessel by taking the measurements in centimetres

$$\begin{aligned} &= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 1\,000\,000 \text{ cm}^3 \\ &= 1\,000\,000 \text{ ml (as } 1 \text{ ml} = 1 \text{ cm}^3) \\ &= \frac{1\,000\,000}{1\,000} \text{ l (as } 1 \text{ l} = 1000 \text{ ml)} \\ &= \underline{\underline{1\,000 \text{ l}}} \end{aligned}$$

Whether the measurements are in m or cm the same volume is shown

$$\therefore 1 \text{ m}^3 = 1\,000 \text{ l}$$

1 cubic metre is a volume of 1 000 l

Example 5

The measurements of a water tank built in a house are, length 1.5 m, breadth 1 m height 1 m. Find in litres the volume of water it can hold.

$$\begin{aligned} \text{The volume of the tank} &= 1.5 \text{ m} \times 1 \text{ m} \times 1 \text{ m} \\ &= 1.5 \text{ m}^3 \\ 1 \text{ m}^3 &= 1\,000 \text{ l} \\ \therefore 1.5 \text{ m}^3 &= 1.5 \times 1\,000 \text{ l} \\ &= \underline{\underline{1\,500 \text{ l}}} \end{aligned}$$

Example 6

A tank the shape of a cube, the length of a side of which is 1 m is filled with water. 200 l of water from the tank is used daily. For how many days will the water in the tank be sufficient?

$$\begin{aligned} \text{The volume of water in the tank} &= 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} \\ &= 1 \text{ m}^3 \\ &= 1000 \text{ l} \\ \text{The volume of water used each day} &= 200 \text{ l} \end{aligned}$$

$$\begin{aligned} \therefore \text{The number of days for which the water will be sufficient} &= \frac{1000}{200} \\ &= \underline{\underline{5 \text{ days}}} \end{aligned}$$

Exercise 9.3

1. Fill in the blanks of the table given below.

cm^3	ml	l	m^3
50 000	50 000	50	0.05
-----	-----	---	1
-----	-----	2 000	-----
-----	4 600 000	-----	-----
-----	-----	-----	4

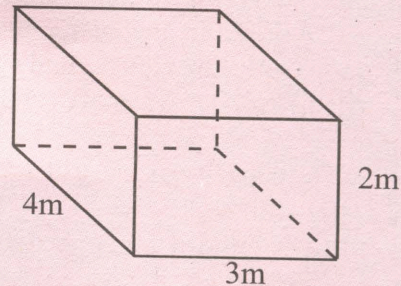
2. Express the following volumes given in l and ml , in m^3

- (i) 2 500 l (ii) 3 000 l (iii) 800 l (iv) 200 l
 (v) 50 l (vi) 1 l (vii) 1 500 ml (viii) 25 000 ml

3. Find in litres the volumes given below.

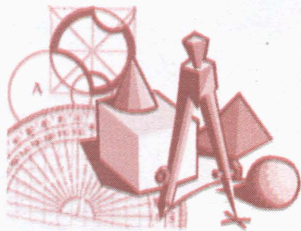
- (i) 2 m^3 (ii) 10 m^3 (iii) 0.5 m^3 (iv) 1.2 m^3

4. A tank, the measurements of which are shown in the given diagram is filled with water. A pipe which is installed to empty the tank, takes 1 minute to empty 300 l of water. Find the time needed to empty the water in the tank.



5. (i) 4 l of milk is obtained from a cow in a day. What is the average quantity of milk obtained from the cow in 100 days?
 (ii) If a cow during her life time gives milk in the above manner five times after giving birth to calves, what is the quantity of milk she gives during her life time?
6. (i) The capacity of a large container of milk is 3 m^3 . How many litres of milk is there when it is full of milk?
 (ii) If that quantity of milk was obtained from 750 cows in a farm, what is the average number of litres collected from one cow?
 (iii) How many packets of milk each of 100 ml can be obtained from that quantity of milk?

7. (i) A bee can collect 1 ml of pollen in one thousand turns. What is the quantity of pollen the bee brings in one turn?
(ii) Assuming that one bee brings pollen 1 000 times, find how many bees are needed to collect 1 l of pollen.
(iii) If one bee collects 2 ml of pollen in its life time, find the number of turns in which the bee should bring pollen to collect 2 ml?
8. Amali who went to the tap in the school, drank water in handfuls in between talking to her friend. She drank water with one hand six times in 5 minutes.
(i) If one handful of water is equal to 10 ml, what is the volume of water she drank in 5 minutes?
(ii) If the speed of the flow of water through the tap is 2 l a minute, what is the volume of water that came out in 5 minutes?
(iii) How many millilitres of water went waste?
(iv) Calculate the percentage of water wasted.
(v) If Amali drank water with both hands and that quantity was equal to 30ml, how many times should she use the two hands to drink the same volume of water as before?
(vi) If Amila drank water without talking to her friend, she could finish drinking the water in 1 minute. In such a case, find the number of millilitres that would go waste.
(vii) Propose ways of drinking water with minimum waste.
9. The capacity of a vessel is 2 m^3 . Find in,
(i) litres (ii) millilitres (iii) cm^3
the volume of water needed to fill the tank
10. The internal length, breadth and height of a water tank which a local Authority owns are 5 m, 5 m and 3 m respectively. When this tank is filled, the volume of water stored is just sufficient to supply 60 houses for a day.
(i) Find the internal volume of the water tank.
(ii) Find the capacity of the tank in litres.
(iii) What is the average number of litres a house uses in a day?
(iv) If the local authority spends Rs 1 for every 100 l of water for purification, distribution and maintenance, what is the sum of money the local authority spends on the quantity of water a house uses for a month?



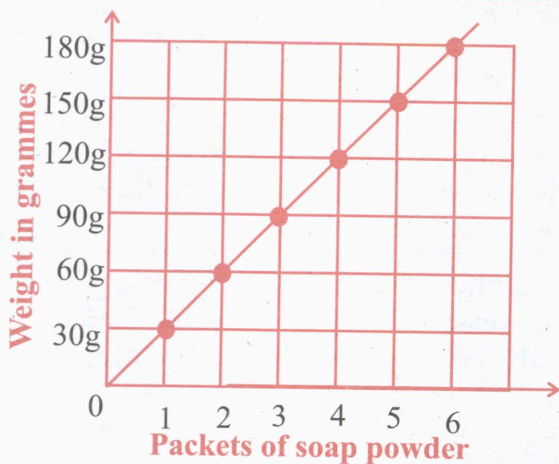
Direct Proportion

10

By studying this chapter you will be able to achieve the following competencies.

- ★ Identifying direct proportions.
- ★ Doing calculations by applying direct proportions.
- ★ Solving problems related to direct proportions by using the unitary method.
- ★ Solving problems with conversions of foreign currency.

10.1 Features of direct proportion



This graph represents how a number of packets of a certain variety of soap powder varies with its weight.

The weight of 2 packets of soap powder

= 60 g

The weight of 5 packets of soap powder

= 150 g

The ratio of the number of packets of soap powder

= 2 : 5

The ratio of the relevant weight

= 60 : 150

When expressed in the simplest form

= 2 : 5

Accordingly

The ratio of the number of packets of soap powder

=

The ratio of the corresponding of weights

If the ratio of two quantities is equal to the ratio of two other quantities relevant to the first two quantities, such a relation is called a proportion.

Now let us consider the proportion discussed below

	The sum of money borrowed (Rupees)	Interest for a year (Rupees)
A	1 000	120
B	2 000	240
C	3 000	360
D	4 000	480
E	5 000	600

Sum of money that A and C borrowed

A	- 1 000
C	- 3 000

Interest paid

Rs 120
Rs 360

The ratio of the sum of money borrowed by A and C

$$1\ 000 : 3\ 000$$

$$1 : 3$$

The ratio of the interest paid by A and C

$$120 : 360$$

$$1 : 3$$

When the values that belong to two different quantities are compared as above, if the ratio of the two values of the first quantity is equal to the ratio any two values of the second quantity relevant to the values of the first quantity, the relation between the two quantities is known as a **direct proportion**.

When one quantity out of two quantities which are in direct proportion increases, the other quantity too increases in the same ratio in relation to the first. Also, when one quantity decreases, the other quantity too decreases in the same ratio.

The amount of money	Interest	Constant
Rs 1 000	Rs 120	$\frac{120}{1000} = 0.12$
Rs 3 000	Rs 360	$\frac{360}{3000} = 0.12$

The ratio of two quantities which are directly proportional is a constant

Example 1

Fill in the box :

$$2 : 3 = \square : 12$$

$$3 \times 4 = 12$$

$$2 \times 4 = 8$$

$$\therefore \underline{\underline{2 : 3 = 8 : 12}}$$

Exercise 10.1

- Find suitable values for the blank boxes in the following proportions.
 - $3 : 4 = 12 : \square$
 - $2 : 5 = \square : 20$
 - $\square : 3 = 16 : 12$
 - $5 : \square = 25 : 20$
- Are the two quantities in each of the following situations directly proportional or not?
 - The number of metres of cloth and its value.
 - The number of days a daily paid employee works and the salary that should be paid.
 - The radius and the circumference of a circle.
 - The area and the length of a side of a square.
 - The number of boxes of coloured pencil of the same brand and the number of pencils in the boxes.
 - The time taken to pay an amount of money borrowed at a certain rate of interest and the interest that has to be paid.
 - The speed of a motor vehicle and the time taken to cover a certain distance.
 - The number of persons engaged in a certain job of work and the time taken to complete it.
- The relationship between the radius of a circle and its area is shown in the table given below

Radius of a circle	Area of the circle
7 cm	154 cm^2
14 cm	616 cm^2
21 cm	1386 cm^2
28 cm	2464 cm^2

Examine the above information and show giving reasons whether the two quantities are directly proportional or not.

10.2 Unitary Method

The price of 100 g of biscuits is Rs 14. What is the price of 350 g of biscuits of the same brand?

How two pupils solved the above problem is shown below.

Malitha	Dileepa
The price of 100 g of biscuits = Rs 14	The price of 100 g of biscuits = Rs 14
The price of 300 g of biscuits = Rs 14 × 3 = Rs 42	The price of 1g of biscuits = $\frac{14}{100}$
The price of 50 g of biscuits = Rs 14 ÷ 2 = Rs 7	The price of 350 g of biscuits = $\frac{14}{100} \times 350$ = <u>Rs 49</u>
The price of 350 g of biscuits = Rs 42 + 7 = <u>Rs 49</u>	

Which in your opinion is the short and easy method out of the two methods above? It is not always easy to divide a unit into parts. As such the first method is rather difficult. It will be seen that the second method can be applied very often.

The method of solving a problem taking the unit value as the base is known as the unitary method.

If the value of a few units is known, the value of one unit and then the value of any number of units can be calculated using the unitary method.

Example 2

A train travels at a speed of 96 kilometres an hour. What is the distance it travels in 5 minutes?

(i) In the unitary method

$$\begin{aligned}
 \text{The distance it travels in 1 hour} &= 96 \text{ km} \\
 \text{The distance it travels in 60 minutes} &= 96 \text{ km} \\
 \text{The distance it travels 1 minute} &= \frac{96}{60} \\
 \text{The distance it travels in 5 minutes} &= \frac{96}{60} \times 5 \\
 &= \underline{\underline{8 \text{ km}}}
 \end{aligned}$$

(ii) Using proportions

Time	Distance
60 min	96 km
5 min	x
$60 : 5$	$= 96 : x$

$$\begin{aligned} \frac{60}{5} &= \frac{96}{x} \\ 60x &= 96 \times 5 \\ x &= \frac{96 \times 5}{60} \\ x &= \underline{\underline{8 \text{ km}}} \end{aligned}$$

Example 3

An article bought for Rs 300 is sold keeping a profit of 25%. Find its selling price.

(i) In the unitary method

The selling price of an article bought for Rs 100 = Rs 125

The selling price of an article bought for Rs 1 = Rs $\frac{125}{100}$

The selling price of an article bought for Rs 300 = Rs $\frac{125}{100} \times 300$
 = Rs 375

(ii) Using proportions

Buying price	Selling price
100	125
300	x
$100 : 300$	$= 125 : x$
$\frac{100}{300}$	$= \frac{125}{x}$
$100x$	$= 300 \times 125$
x	$= \frac{300}{100} \times 125$
	$= \underline{\underline{\text{Rs } 375}}$

Exercise 10.2

Part A

Solve the following problems using the unitary method.

1. The price of 3 metres of shirting is Rs 675. Find the price of 5 m of shirting of the same kind.
2. A motor vehicle travels a distance of 48 km with 4 litres of petrol. Find the distance it will travel with 10 litres of petrol.

3. If a motorcycle travelling at a uniform speed covers a distance of 1.5 km in 5 minutes; find the distance it covers in 12 minutes.
4. If the price of a bag of 25 apples is Rs 300, what is the price of 10 apples of the same kind?
5. If a discount of Rs 40 is given for an item, the marked price of which is Rs 500 what is the discount that will be given for an item, whose marked price is Rs 750?
6. The weight of a steel rod of length 6 m is 1.8 kg. Find the weight of a steel rod of the same brand of length 5 m.
7. Given below is a bill issued by a shop to 'Kusal' for the goods he has bought. In addition to it, a list of goods bought from the same shop by 'Dileepa' is also given below. Complete Dileepa's bill.

Kusal's bill

4 kg rice	Rs 212
2 kg sugar	Rs 160
3 packets of flour	Rs 795
5 cases of soap	Rs 150
2 packets, of margarine of 100g	Rs 48
5 Exercise books	Rs 60
	Rs 1 425

Dileepa's list of goods

3 kg rice	= -----
3 kg sugar	= -----
2 packet of flour	= -----
3 cases of soap	= -----
1 packet, of margarine of 100 g	= -----
12 Exercise books	= -----

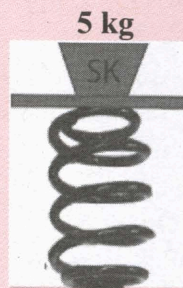
8. A train takes 24 minutes to travel 32 km. Find the speed of the train in kilometres per hour.
9. A train covers 96 km in one hour. Find the time the train takes to travel 288 km.
10. A motor vehicle travels 120 km in $2\frac{1}{2}$ hours. What is its speed in kilometres per hour?

Part B

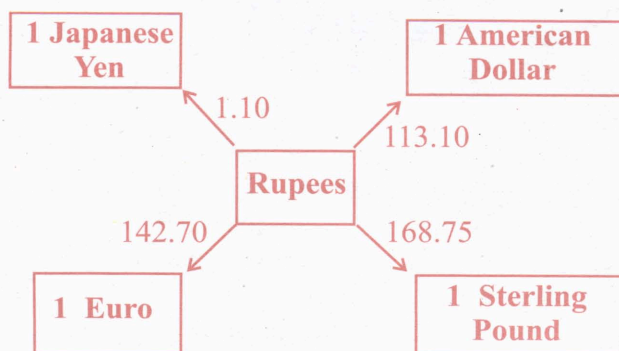
Solve the following problems applying direct proportions.

11. If the wages for 8 masons for one day is Rs 4 800, what will be the wages for 5 masons for one day?
12. An article is bought for Rs 400 and is sold keeping a profit of 12% .What is its selling price?
13. By selling an article for Rs 565 a trader makes a profit of 13%. What is its cost price?
14. The monthly instalment for a loan of Rs 36 000 obtained from a bank at 12% simple interest to be paid in 3 years is Rs 1 360. If from the same bank for the same simple annual interest rate another loan of Rs 108 000 is obtained to be paid in 3 years, what will be the value of an instalment?

15. The weight of 16 cm^3 of a metal is 24 g. Find the weight of 20 cm^3 of the same metal.
16. When the height of the mast of a ship is 12 m, the height of the mast of a model of the ship made to a scale is 9 cm. Accordingly, if the length of the ship is 24 m, find the length of the model of the ship.
17. The spring of a spring balance extends to a length of 2.6 cm when a weight of 8 Newtons is upheld. Find the length to which the spring extends when the weight is 3.6 Newtons.
18. A machine in a factory which produces aerated water can fill 840 bottles in 6 hours. If the machine operates for 5 hours, find the number of bottles that will be filled with aerated water.
19. When a weight of 5 kg is placed on a coil it contracts by 25 mm. By how many mm will the coil contract if a weight of 12.5 kg is placed on it?
20. The real distance between two cities shown 3 cm apart on a map is 5 km. Find the real distance between two other cities shown 12 cm apart on the map.



10.3 Foreign Currency.



The relationship between the unit of currency used in some foreign countries and Sri Lanka is shown above. We know that each country has its own unit of currency. The relationship between the unit of currency of one country and the unit of currency of another country is known as the rate of exchange. There is a difference between the number of Rupees paid for an American Dollar in Sri Lanka and the number of Yens paid in Japan. Those who tour foreign countries, convert to American Dollars the currency of one's own country, and when entering another country convert those to the currency of the country one enters. When converting so, the rate of exchange prevailing at the time will be used. Given below are some rates of exchange used to convert foreign currency in Sri Lanka. These rates change very often.

Rates of exchange (on a certain day)

Foreign Currency Unit	Value in Sri Lankan Rupees
1 American Dollar	Rs 113.10
1 Bahrain Dinar	Rs 282.10
1 Euro	Rs 142.70
1 Japanese Yen	Rs 1.10
1 Oman Riyal	Rs 273.40
1 Pakistan Rupee	Rs 1.40
1 Sterling Pound	Rs 168.75
1 Australian Dollar	Rs 76.25

Find the present rates of exchange from a News paper or from any other source

Monetary conversions

Example 4

The salary for a month of a Sri Lankan working abroad, is 400 American Dollars. What is his salary in Sri Lankan rupees?

$$\begin{aligned}
 1 \text{ American Dollar} &= \text{Rs } 113.10 \\
 400 \text{ American Dollars} &= \text{Rs } 113.10 \times 400 \\
 &= \underline{\underline{\text{Rs } 45\,240}}
 \end{aligned}$$

Example 5

A housemaid who hopes to go abroad, forwarded a sum of Rs 56 550 to a bank asking for it to be converted to American Dollars. How many dollars will she get?

$$\begin{aligned}
 \text{Rs } 113.10 &= \text{American dollar } 1 \\
 \text{Rs } 56\,550 &= \text{American Dollars } \frac{56\,550.00}{113.10} \\
 &= \underline{\underline{\text{American Dollars } 500}}
 \end{aligned}$$

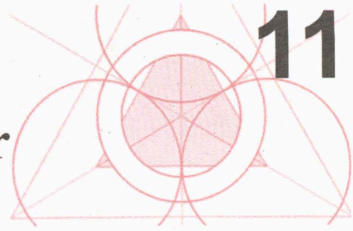
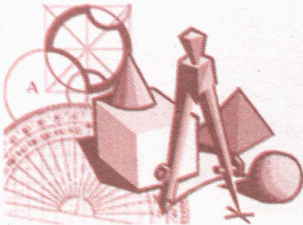


Exercise 10.3



Use the rates of exchange in the table given earlier to do the following exercise.

1. An international organization donates a sum of Euros 500 for a project in Sri Lanka. Find its value in Sri Lankan rupees.
2. Dileepa who is working abroad sent Yens 50 000 to his mother in Sri Lanka to be deposited in his bank account. His mother got the money converted to Sri Lankan Rupees and deposited it in to his bank account, Find the sum of money she deposited in Rupees.
3. A house - maid who returned to Sri Lanka from a foreign country converted Sterling Pounds 5000 from her NRFC account to Sri Lankan rupees. Find the amount of money she obtained.
4. A person got Rs 71 350 converted to Euros from a bank. Find the amount of Euro he obtained.
5. The selling price of an American Dollar is Rs 113.10. How many American Dollars can a person buy for Rs 282 750 from a bank?
6. 'Sugath' who leaves to Australia on a scholarship, got Rs 381 250 converted to Australian Dollars. How many Dollars did he get?
7. How many Pakistan rupees can be bought for 70 Sri Lankan rupees?
8. What is the value of a saree in Sri Lankan rupees, which was bought in Pakistan for Pakistan Rs 200?



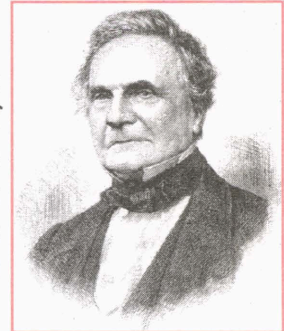
Calculator

By studying this chapter you will be able to achieve the following competencies.

- ★ Identifying an ordinary calculator and its operations.
- ★ Manipulating the four basic mathematical operations using the calculator.
- ★ Using the calculator for repeated operations of Addition, Subtraction, Multiplication and Division.
- ★ Adding or subtracting a constant and multiplying or dividing by a constant.
- ★ Using the calculator for operations involving integers, fractions, decimals, percentages.
- ★ Using the calculator to find the square or the square root of a number.
- ★ Using the calculator to generate number patterns and thereby appreciating the results.

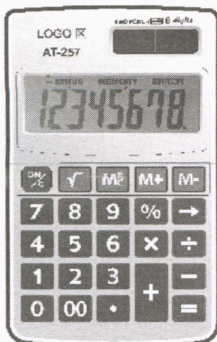
11.1 Let us identify a calculator

A calculator helps us to perform a mathematical operation accurately, quickly and easily. There are two types of calculators namely Ordinary Calculator and Scientific Calculator. Used for different purposes though a calculator can be used to do certain actions which a human brain performs, the creative abilities that a human brain possess are not seen in any calculator.



Charles Babbage invented the first calculator in 1833

The ordinary calculator



← Display screen

← Keyboard

There can be small differences in the keyboards as many organizations produce calculators. The results obtained when keys are activated appear on the display screen.

11.2 Identifying the keyboard of a calculator

Key	Function												
ON	The calculator gets activated.												
OFF	The calculator stops operations												
<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>7</td><td>8</td><td>9</td></tr> <tr><td>4</td><td>5</td><td>6</td></tr> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>0</td><td>.</td><td></td></tr> </table>	7	8	9	4	5	6	1	2	3	0	.		The digit or the decimal point on the activated key appears on the display screen.
7	8	9											
4	5	6											
1	2	3											
0	.												
=	The results of the mathematical operations appear on the display screen.												
CE CL	Erases any digit entered by mistake after a mathematical operation												
AC	Erases everything on the display screen												
+ - × ÷	Does the mathematical operation indicated on the key												

Examples of using keys $+$ $-$ \times \div for mathematical operations are given below

Example 1

Simplify	Process	Answer
(i) $135 + 87$	ON 1 3 5 + 8 7 =	222
(ii) $521 - 97$	ON 5 2 1 - 9 7 =	424
(iii) 735×49	ON 7 3 5 \times 4 9 =	36 015
(iv) $1078 \div 98$	ON 1 0 7 8 \div 9 8 =	11
(v) 27.5×57	ON 2 7 . 5 \times 5 7 =	1 567.5

Exercise 11.1

Use the calculator for the following calculations.

1. Simplify

(i) $1\ 007 + 75$

(ii) $75 + 27 - 12$

(iii) $2.75 + 7.2$

(iv) $1\ 003 - 97$

(v) 380×227

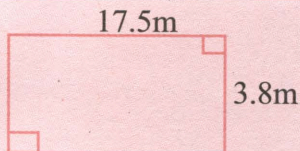
(vi) 0.005×47

(vii) $512 - 3.2$

(viii) $\frac{43.75}{35}$

2. ON 5 2 + 5 CE 8 7 = . What is the number that will appear on the display screen when these keys are operated?

3.



The length of a rectangular paddy field is 17.5 m and its breadth is 3.8 m. Find its area.

4. If Rs 26 250 is divided equally among 35 pupils, how much will each get?
5. The monthly salary of 'Raja' is Rs 8 295.50. What is his annual salary?
6. "Gayani" operated the calculator and obtained the answer of 37.5×15 as 56.25. Is her answer correct or incorrect? If incorrect, where has she gone wrong in your opinion?

11.3 Repeated Mathematical Operations.

Addition of one number to another number repeatedly or subtraction of one number from another repeatedly can be done easily by a calculator.

Example 2

Repeat the addition of 3 to 5

For this, use the keys as shown below.

$\boxed{\text{ON}} \boxed{5} \boxed{+} \boxed{3}$ then the key $\boxed{=}$ the required number of times.

The result will appear as 8, 11, 14, 17 -----

In some calculators the keys should be operated as $\boxed{\text{ON}} \boxed{5} \boxed{+} \boxed{+} \boxed{3}$ and the $\boxed{=}$ key the required number of times. Still there are some other calculators in which the numbers have to be changed as shown below.

$\boxed{\text{ON}} \boxed{3} \boxed{+} \boxed{5} \boxed{=}$ or $\boxed{\text{ON}} \boxed{3} \boxed{+} \boxed{+} \boxed{5} \boxed{=}$

Now operate the way shown in the following examples and observe the result.

Example 3

Subtract 3 repeatedly from 45

$\boxed{\text{ON}} \boxed{4} \boxed{5} \boxed{-} \boxed{3}$ activate the $\boxed{=}$ key again and again.

The result will appear as 42, 39, 36, ---

Example 4

Multiply 5 by 2 repeatedly

Operate the keys as shown below.

$\boxed{\text{on}} \boxed{5} \boxed{\times} \boxed{2}$ and $\boxed{=}$ again and again

or $\boxed{\text{on}} \boxed{2} \boxed{\times} \boxed{5}$ and $\boxed{=}$ again and again

The result will be 10, 20, 40 ---

Example 5

Divide 200 by 2 repeatedly.

Operate the keys as shown below.

ON 2 0 0 ÷ 2 and = again and again

The result will be 100, 50, 25, ---

11.4 Mathematical operations with a constant

Operate a calculator and comprehend the following examples

Example 6

(i) Addition of a constant

Add 20 to each of these numbers : 38, 70, 135

ON 2 0 + + 3 8 = ON 3 8 + + 2 0 =
7 0 = .or 7 0 =
1 3 5 = 1 3 5 =

Answers = 58, 90, 155

(ii) Subtracting a constant

Subtract 135 from each of the following numbers; 1 200, 1 570, 1 800, 2 000

ON 1 2 0 0 - - 1 3 5 =
1 5 7 0 =
1 8 0 0 =
2 0 0 0 =

Answers = 1 065, 1 435, 1 665, 1 865

(iii) Multiply the numbers 5, 15, 35 by 2

ON 2 × × 5 =
1 5 =
3 5 =

Answers = 10, 30, 70

(iv) Divide the numbers 200, 160, 70 by 2

ON 2 0 0 ÷ ÷ 2 =
1 6 0 =
7 0 =

Answers = 100, 80, 35



Exercise 11.2



Obtain answers using repeated mathematical operations.

- Add 5 to 70, three times repeatedly and note the results.
- Write four answers when Rs 35 is subtracted repeatedly from Rs 600
- Find the value of $512 \times 2 \times 2 \times 2$
- Obtain the result of $5478 \div 2 \div 2 \div 2$
- Given below are the marks a group of pupils obtained for a work sheet out of 20. Convert these to marks out of 100.
6, 8, 10, 14, 16, 18
- Apply repeated mathematical operations and find the next 3 terms of the following number patterns.
 - 5, 7, 9, 11 -----
 - 58, 53, 48, 43 -----
 - 2, 6, 18, -----
 - 400, 200 -----
- The monthly salary in rupees of eight persons working in an office are given below.
8 000, 9 200, 11 500, 11 800, 12 000, 15 200, 17 500, 20 000
add Rs 675 to each salary and prepare a new paysheet.

11.5 Converting fractions to decimals

Observe and comprehend the examples of converting fractions to decimals, given below.

Example 7

Fraction	Process	Decimals
(i) $\frac{1}{8}$	ON 1 \div 8 =	0.125
(ii) $\frac{2}{3}$	ON 2 \div 3 =	0.6666----
(iii) $\frac{5}{12}$	ON 5 \div 12 =	0.41666 ----

11.6 Simplification of integers.

The key $\boxed{+/-}$ is used to show integers.

We know that the symbol '+' is not essential to show positive integers; but in the case of negative integers the symbol '-' is essential.

After operating the keys ON $\boxed{7}$ if the key $\boxed{+/-}$ is operated repeatedly you will obtain 7, -7, 7 - 7, 7 -----

Observe the simplification procedure and the answers of the examples given below.

Example 8

Simplify :

(i) $5 + (-27)$	ON	5	+	2	7	+/-	=	-22			
(ii) $(-2) - (-1)$	ON	2	+/-	-	1	+/-	=	-1			
(iii) $(-12) \div (-2)$	ON	1	2	+/-	\div	2	+/-	=	6		
(iv) $(-8) \times 25$	ON	8	+/-	\times	2	5	=	-200			
(v) $\frac{-3 \times 15}{-9}$	ON	3	+/-	\times	1	5	\div	9	+/-	=	5

Exercise 11.3

1. Indicate in the boxes the procedure of converting $\frac{3}{4}$ to a decimal.

ON

2. Using the calculator convert the following fractions to decimals.

(i) $\frac{1}{4}$

(ii) $\frac{3}{8}$

(iii) $\frac{8}{15}$

(iv) $\frac{13}{20}$

(v) $\frac{8}{25}$

(vi) $\frac{1}{3}$

(vii) $\frac{1}{6}$

(viii) $\frac{5}{11}$

3. Give the value of $\frac{22}{7}$ to three places of decimals.

4. Simplify :

(i) $125 + (-17)$

(ii) $(-12) - (-27)$

(iii) $(-36) \div (-4)$

(iv) $2.5 \times (-10)$

(v) $(-7) \times (-8) \div (-28)$

'Pathima' says that the value of $\frac{3}{7}$ correct to 3 decimal places is 0.429. Use the calculator and give reasons as to why her statement is true.

When doing mathematical operations using the ordinary calculator the BODMAS rule should be followed.

11.7 Using memory keys

Key	Function
M+	Storing data in memory
MR RM	Recalling data from memory to the screen
MC CM	Removes from memory
M-	Subtract a value from the memory

Immediately after solving a problem M should be removed from the screen.

Example 9

Simplify : $\frac{45}{7+2}$

ON 7 + 2 = M+ 4 5 ÷ MR =

To remove M at the end operate

MC

Example 10

Calculate the sum of money needed to buy 2.25 m of cloth at the rate of Rs70 a metre and 1.5m of lace at the rate of Rs 17.50 a metre.

ON 7 0 × 2 . 2 5 = M+ 157.50

1 7 . 5 0 × 1 . 5 = M+ 26.25

MR 183.75

Answer = Rs.183.75

Operate MC

Exercise 11.4

1. Use the memory key and simplify the following.

(i) $\frac{17}{7-2}$

(ii) $\frac{5+3}{4 \times 2}$

(iii) $\frac{23 \times 24}{5.8-4.6}$

(iv) $\frac{37 \times 8.4}{57 \div 19}$

(v) $\frac{17.5+3.5}{12.8-2.3}$

2. Use the memory key and prepare a bill for the following. Calculate the sum of money needed to pay the bill.

Material	Quantity	Price of a unit (Rs)
Sand	3 cubes	800
Cement	12 bags	750
Lime	2 bags	85

11.8 Percentages, Powers and Square root.

% Percentage key

Example 11

1. What is 5% of Rs 80?

ON Answer = Rs 4.00

2. Express $\frac{3}{5}$ as a Percentage

ON Answer = 60%

The square of a number

The key generates second power of a number.

Example 12

Find: 7^2

ON Answer = 49

If the key is for operate as shown below.

The square root of a number

Use the key to obtain the square root of a number.

Example 13

Find $\sqrt{289}$

ON = 17



Exercise 11.5

Use the calculator for the following calculations

1. Find the value of,

(i) 8% of Rs 1 200

(ii) 12.5% of 3 600

2. 18 out of 25 students who appeared for an examination have got merit passes. Express this as a percentage.

3. Express the fractions given below as percentages.

(i) $\frac{2}{5}$

(ii) $\frac{3}{4}$

(iii) $\frac{20}{40}$

(iv) $\frac{11}{5}$

4. Find the value of :

(i) 9^2

(ii) 35^2

(iii) 1.5^2

(iv) 2.5^2

(v) 2.75^2

5. Find the value of :

(i) $\sqrt{25}$

(ii) $\sqrt{169}$

(iii) $\sqrt{324}$

(iv) $\sqrt{12.25}$

(v) $\sqrt{27.04}$

6. Simplify: $\frac{22}{7} \times 1.5^2 \times 2.8$

7. Simplify: $\frac{5.12 \times \sqrt{1.44}}{0.6}$

8. Round off $\frac{\sqrt{3}}{\sqrt{2}}$ to the first decimal place.

9. (i) Get 1, 2, 3, 4, 5, 6, 7, 9 on to the screen and operate.

\times 9 =

(ii) Get 1, 2, 3, 4, 5, 6, 7, 9 on to the screen again and operate

\times 18 =

(iii) Observe the results that you obtained and write what steps that you should take to obtain 55555555 on the screen.

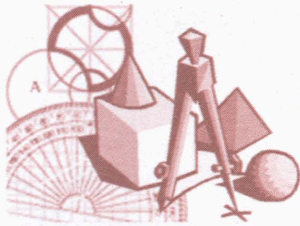
10. ★ Do the following simplifications with your calculator.

★ Turn your calculator up side down and observe every solution you get. Read each English term that you see on the screen.

(i) 3 5 \times 1 0 0 + 4 =

(ii) 3 5 \times 1 0 0 0 + 6 =

(iii) 2 7 \times 2 0 0 0 - 2 = \div 7 =

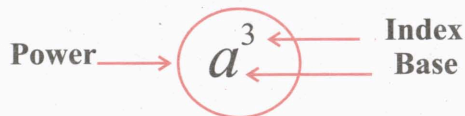


Indices and Logarithms 12

By studying this chapter you will be able to achieve the following competencies

- ★ Multiplying and dividing of powers with the same base.
- ★ Simplifying indices of a power with a power.
- ★ Identifying the zero index and negative indices and using them in calculations.
- ★ Using laws of indices in calculations.
- ★ Identifying the logarithm of a number
- ★ Writing an expression with an index as an expression in logarithms.
- ★ Writing an expression in logarithms as an expression in index form.

What you learnt about indices in previous grades can be summarized as follows.



1. Writing powers in the expanded form.

(i) $5^3 = 5 \times 5 \times 5 = 125$ (ii) $b^5 = b \times b \times b \times b \times b$

(iii) $(ab)^3 = a^3 \times b^3 = a \times a \times a \times b \times b \times b$

$$(ab)^3 = ab \times ab \times ab \\ = a \times a \times a \times b \times b \times b$$

2. Expressing in index form

(i) $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$ (ii) $x \times x \times x \times x = x^4$

(iii) $3 \times 3 \times y \times y \times y = 3^2 \times y^3 = 9y^3$

3. Expressing a power of a product as a product of powers

(i) $(ab)^3 = a^3 \times b^3 = a^3 b^3$ (ii) $(5y)^2 = 5^2 \times y^2 = 25y^2$

4. Expressing a given product of powers as a power of a product

(i) $16p^4 = 2^4 \times p^4 = (2p)^4$ (ii) $q^2 \times n^2 = (qn)^2$

5.

Even powers	Odd powers
$(-2)^4 = -2 \times -2 \times -2 \times -2 = +16$	$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$
$(-3)^4 = -3 \times -3 \times -3 \times -3 = +81$	$(-3)^5 = -3 \times -3 \times -3 \times -3 \times -3 = -243$
An even number of power of a negative number is a positive value	An odd number of power of a negative number is a negative value

12.1 Multiplication of powers with equal bases

Example 1

$$\begin{aligned} 3^3 \times 3^4 &= (3 \times 3 \times 3) (3 \times 3 \times 3 \times 3) \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^7 \end{aligned}$$

Similarly ;

$$\begin{aligned} 3^3 \times 3^4 &= 3^{3+4} \\ &= 3^7 \end{aligned}$$

Expression	Simplify by expanding	Simplifying using indices
1. $3^3 \times 3^2$ 2. $m^4 \times m^2$ 3. $p^3 \times q^2 \times p^5 \times q$	$3 \times 3 \times 3 \times 3 \times 3 = 3^5$ $m \times m \times m \times m \times m \times m = m^6$ $(p \times p \times p) \times (q \times q) \times (p \times p \times p \times p \times p) \times q$ $= (p \times p \times p \times p \times p \times p \times p \times p) \times (q \times q \times q)$ $= p^8 \times q^3$	$3^{3+2} = 3^5$ $m^{4+2} = m^6$ $p^{3+5} \times q^{2+1} = p^8 q^3$

Accordingly, the product of two powers with equal base 'a' and indices 'm' and 'n' can be written as

$$a^m \times a^n = a^{m+n}$$

i.e., When two powers of equal base are multiplied, their indices are added, but the base will remain the same.

Exercise 12.2

1. Fill in the blanks.

(i) $3^2 \times 3^6 = 3^{\square}$

(ii) $a^3 \times a^8 = a^{\square}$

(iii) $5^4 \times 8^2 \times 5^2 = 5^{(\square+2)} \times 8^2$

(iv) $p^3 \times q^4 \times p^6 \times q^3 = p^{(3+\square)} \times q^{(\square+\square)}$

(v) $a^4 \times b^5 \times a^6 \times b^2 = a^{(\square)} \times b^{(\square)}$

(vi) $2^2 \times c^4 \times 2^4 \times c^5 = 2^{(\square+\square)} \times c^{(\square+\square)} = 2^{\square} \times c^{\square}$

(vii) $4^{\square} \times k^1 \times 4^5 \times k^{\square} = 4^7 \times k^5$

(viii) $\left(\frac{1}{3}\right)^5 \times \left(\frac{1}{3}\right)^{\square} = \left(\frac{1}{3}\right)^9$

(ix) $x^{\square} \times 7^2 \times x^4 \times 7^4 = x^6 \times 7^{\square}$

(x) $(0.2)^3 \times (0.2)^5 \times (0.2)^{\square} = (0.2)^{20}$

2. Write 5 pairs of values separately for x and y to satisfy the following relation

$$a^x \times a^y = a^{50}$$

3. Fill in the blank squares with suitable numbers.

$$\begin{array}{ccc}
 5^6 \times 5^2 \times 5^8 \times 5^{\square} & & x \times x^{\square} \\
 \parallel & & \parallel \\
 5^8 \times 5^{\square} = 5^{20} = 5^{\square} \times 5^{\square} & & x^9 \times x^{\square} = x^{\square} = x^6 \times x^{12} \\
 \parallel & & \parallel \\
 5^4 \times 5^3 \times 5^{\square} & & x^7 \times x^{\square}
 \end{array}$$

12.2 Division of powers with equal bases

Example 2

Simplify $3^4 \div 3^2$

$$\begin{aligned}
 3^4 \div 3^2 &= \frac{3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \\
 &= \underline{\underline{3^2}}
 \end{aligned}$$

Simplify $c^8 \div c^3$

$$\begin{aligned}
 &= \frac{c \times c \times c \times c \times c \times \cancel{c} \times \cancel{c} \times \cancel{c}}{\cancel{c} \times \cancel{c} \times \cancel{c}} \\
 &= \underline{\underline{c^5}}
 \end{aligned}$$

In the above examples, examine that the same answer can be obtained by subtracting the index of the denominator from the index of the numerator.

$$3^5 \div 3^2 = 3^{5-2} = \underline{\underline{3^3}}$$

$$c^8 \div c^3 = c^{8-3} = \underline{\underline{c^5}}$$

Accordingly the division of two powers with the same base 'a' and indices 'm' and 'n' can be shown as follows

$$a^m \div a^n = a^{m-n}$$

or

$$\frac{a^m}{a^n} = a^{m-n}$$

ie, when two powers of equal base are divided, the index of the divisor is subtracted from the index of the dividend, but the base will remain the same

If the value of 'n' is greater than the value of 'm', then 'm-n' will give a negative index. (This will be explained in the next section)

Exercise 12.2

1. Fill in the blanks

(i) $5^7 \div 5^3 = 5^{\square}$

(ii) $\frac{x^8}{x^5} = x^{\square}$

(iii) $a^{\square} \div a^3 = a^{10}$

(iv) $\frac{2^{\square} \times 2^4}{2^3} = \frac{2^9}{2^3} = 2^{\square}$

$$(v) \frac{y^5 \times y^{\square} \times y^3}{y^4 \times y^{\square}} = \frac{y^{10}}{y^8} = y^{\square} \quad (vi) \frac{c^{\square} \times c^5}{c^3 \times c^{\square}} = \frac{c^9}{c^{\square}} = c^4$$

2. Fill in the boxes with suitable numbers.

$$\begin{array}{ccc} \frac{3^5 \times 3^8}{3^{\square}} & & \frac{a^4 \times a^3}{a^{\square}} \\ \parallel & & \parallel \\ \frac{3^6 \times 3^{\square}}{3^{10}} = 3^{10} = \frac{3^{12}}{3^{\square}} & & \frac{a^9 \times a^{\square}}{a^8} = a^{\square} = \frac{a^{10}}{a^5} \\ \parallel & & \parallel \\ \frac{3^7 \times 3^{\square}}{3^2 \times 3^{\square}} = 3^{17-\square} & & \frac{a^{\square}}{a^3} \end{array}$$

12.3 Negative indices

Observe the answers obtained in the simplification of $x^2 \div x^5$ by two different methods.

By expanding	By using the laws of indices
$\begin{aligned} x^2 \div x^5 \\ = \frac{\cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times x \times x \times x} \\ = \frac{1}{x^3} \end{aligned}$	$\begin{aligned} x^2 \div x^5 \\ = x^{2-5} \\ = x^{-3} \end{aligned}$

The two answers must be equal. ie, $x^{-3} = \frac{1}{x^3}$

When 'n' is any number, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$

x^{-n} is a power with a negative index when $n > 0$

Example 3

Find the value of 3^{-2}

$$3^{-2} = \frac{1}{3^2} = \frac{1}{\underline{\underline{9}}}$$

Example 4

Find the value of $\frac{1}{4^{-2}}$

$$\frac{1}{4^{-2}} = 4^2 = \underline{\underline{16}}$$

$$\frac{a^{-x}}{b^{-y}} = \frac{b^y}{a^x}$$

Converting a power with a negative index to a power with a positive index is the same as taking the reciprocal of the relevant power.

Exercise 12.3

1. Fill in the blank boxes.

(i) $2^{-5} = \frac{1}{\square}$

(ii) $x^{-2} = \frac{1}{\square}$

(iii) $\square = \frac{1}{3}$

(iv) $2x^{-1} = \frac{2}{\square}$

(v) $\frac{x^{-3}}{y^{\square}} = \frac{y^6}{x^{\square}}$

(vi) $\frac{3}{2x^{-3}} = \frac{3\square}{2}$

2. Simplify the following expressions and express the answer with positive indices.

(i) $\frac{a^{-2} \times b^{-4}}{b^2}$

(ii) $\frac{2^{-3} \times 5^2}{5^{-4} \times 2^4}$

(iii) $\frac{(2x)^3 \times (2x)^{-4}}{(2x)^{-6}}$

(iv) $\frac{8x^2 \times 5y^{-3}}{15x^{-4} \times 2y^5}$

(v) $\frac{3^{-2} \times p^2 \times q^{-2}}{p^{-4} \times q^2}$

(vi) $\frac{c^3 \times m^{-4}}{m^3 \times c^{-3}}$

3. Fill in the blank boxes with suitable values.

(i) $\frac{1}{128} = 2^{\square}$

(ii) $\frac{1}{125} = \square^{-3}$

(iii) $27^{-1} = \left(\frac{1}{3}\right)^{\square}$

(iv) $0.001 = \square^{-3}$

12.4 Null index

When $5^3 \div 5^3$ is simplified using the method of expanding and by using the laws of indices, the following results are obtained.

By expanding	By using the laws of indices
$5^3 \div 5^3$ $= \frac{\cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5}}$ $= 1$	$5^3 \div 5^3$ $= 5^{3-3}$ $= 5^0$

These two answers are equal i.e., $5^0 = 1$

When the base of any power is not zero but the index is zero, its value will be equal to 1.

i.e., when $a \neq 0$,

$$a^0 = 1$$

Exercise 12.4

Simplify :

(i) $x^5 \div x^5$

(ii) $\left(\frac{3}{x^2}\right)^0 \times \frac{x^3}{9}$

(iii) $\frac{(2x)^0 \times x^7}{x^{-2}}$

(iv) $\frac{x^{\frac{1}{2}} \times y^{\frac{2}{3}}}{y^{\frac{1}{3}} \times x^{\frac{1}{2}}}$

(v) $\frac{(xy)^0 \times a^5 \times y^4}{a^{-3} \times y^{-3}}$

(vi) $\frac{m^{+5} \times c^{-2} \times m^{-2}}{c^4 \times m^3 \times c^{-6}}$

12.5 Power of a power

$(2^2)^3$ is an expression of a power of a power. This is, the third power of two to the power 2. This can be simplified by expanding.

$$\begin{aligned} (2^2)^3 &= 2^2 \times 2^2 \times 2^2 \\ &= 2^{2+2+2} \quad (\text{According to the laws of indices}) \end{aligned}$$

$$\therefore (2^2)^3 = 2^6$$

This can be simplified as $2^{2 \times 3} = 2^6$ (By multiplying the indices)

Therefore it can be written as $(2^2)^3 = 2^{2 \times 3}$

Hence if 'a' is any non zero number,

$$(a^m)^n = a^{mn}$$

i.e., When simplifying a power of a power, the indices are multiplied

Exercise 12.5

1. Simplify :

(i) $(3^2)^3$

(ii) $(x^{-2})^3$

(iii) $(y^2)^0$

(iv) $\left(\frac{x^3}{y^2}\right)^2$

(v) $(x^{-3})^{-2}$

(vi) $\left(\frac{a^{-3}}{b^{-2}}\right)^{-3}$

2. Write the suitable number in each blank square

(i) $x^{-10} = (x^{-5})^{\square}$

(ii) $2^{12} = (2^{-6})^{\square}$

(iii) $a^{10} = (a^{\square})^{-\frac{1}{2}}$

(iv) $\frac{(x^3 y^2)^3}{x^7 y^5} = x^{\square} \times y^{\square}$

(v) $\left\{\frac{(0.5) \times (0.5)^6}{(0.5)^8}\right\}^2 = (0.5)^{\square}$

(vi) $\left(\frac{m^3}{n^2}\right)^{-2} = \frac{n^{\square}}{m^{\square}}$

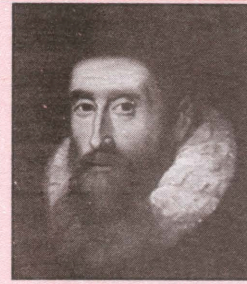
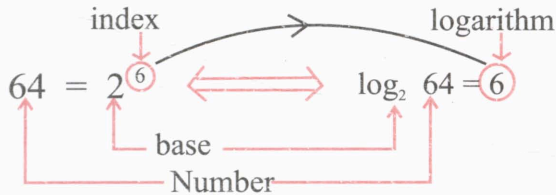
12.6 Logarithms

(a) Introduction of a logarithm of a number

$$64 = 2 \times 2 \times 2 \times 2 \times 2$$

$$\text{This can be written as } 64 = 2^6$$

Then '6' is called the logarithm of 64 to the base 2



JOHN NAPIER
AD 1550 - AD 1617

The Italian mathematician John Napier introduced the idea of logarithm for the first time

(b) Writing an expression given in index form, in logarithmic form

Index form	Logarithmic form	The way of reading the logarithm
$100 = 10^2$	$\log_{10} 100 = 2$	the logarithm of 100 to the base 10 is 2
$32 = 2^5$	$\log_2 32 = 5$	the logarithm of 32 to the base 2 is 5
$49 = 7^2$	$\log_7 49 = 2$	the logarithm of 49 to the base 7 is 2
$a = b^c$	$\log_b a = c$	the logarithm of a to the base b is c
$8 = 2^3$	$\log_2 8 = 3$	the logarithm of 8 to the base 2 is 3
$\frac{1}{8} = 2^{-3}$	$\log_2 \frac{1}{8} = -3$	the logarithm of $\frac{1}{8}$ to the base 2 is -3
$1 = 10^0$	$\log_{10} 1 = 0$	the logarithm of 1 to the base 10 is 0

Generally,

When a number is expressed as a power of another number, the index is the logarithm of the initial number to that base.

ie, if $a = b^c$, then c is the logarithm of ' a ' to the base b ,
This can be expressed as follows

$$\log_b a = c$$

Logarithm to the base 10 is symbolised as lg

$$\log_{10} x = \lg x$$

Here only positive values of a and b are considered

Exercise 12.6

1. Fill in the blanks with suitable values.

(i) $128 = 2^{\square}$ (ii) $0.00001 = \square^{-5}$ (iii) $\frac{1}{256} = 2^{\square}$ (iv) $625 = \square^4$

2. Express the following in logarithmic form

- (i) Logarithm of 1 000 to the base 10
- (ii) Logarithm of 16 to the base 2
- (iii) Logarithm of q to the base p
- (iv) Logarithm of n to the base m

3. Write the following expressions in words.

(i) $\log_3 27$ (ii) $\log_4 1$ (iii) $\log_a b$ (iv) $\log_8 512$

4. Express the following as logarithms.

(i) $128 = 2^7$ (ii) $10000 = 10^4$ (iii) $5 = 5^1$ (iv) $1 = 3^0$

12.7 To convert an expression written in logarithmic form to index form.

An expression written in logarithmic form can be expressed in index form.

Logarithmic form	Index form
(i) $\log_3 243 = 5$	$243 = 3^5$
(ii) $\log_2 1024 = 10$	$1024 = 2^{10}$
(iii) $\log_5 625 = 4$	$625 = 5^4$
(iii) $\log_b a = c$	$a = b^c$

Hence it is understood that a given expression in the form of a logarithm can be converted to index form and also that a given expression in index form can be converted to logarithmic form.

It can be symbolised as

$$a = b^c \Leftrightarrow \log_b a = c$$

Exercise 12.7

1. Express the following in index form.

(i) $\log_5 125 = 3$ (ii) $\log_9 81 = 2$ (iii) $\lg 2 = 0.3010$ (iv) $\lg 0.1 = -1$

2. Fill in the blank boxes with suitable values.

(i) $2^7 = \square \rightarrow \log_2 \square = 7$

(ii) $5^{\square} = \square \rightarrow \log_5 \square = 2$

(iii) $\log_{\square} 125 = 3$ (iv) $\log_2 \square = 5$ (v) $\log_a \square = 4$

3. Fill in the blank boxes with suitable values.

(i) $\log_2 32 = \square$ (ii) $\log_5 25 = \square$ (iii) $\log_x 1 = \square$ (iv) $\log_a a = \square$

4. Fill in the blank boxes.

(i) $\log_{\square} 1000 = 3$ (ii) $\log_{\square} \frac{1}{x} = -1$ (iii) $\log_{\square} \frac{1}{81} = -4$

(iv) $\log_{\square} 0.01 = -2$ (v) $\log_{\square} 16 = 2$ (vi) $\log_{\square} 4^{-2} = -2$

5. Fill in the blank boxes with suitable values.

(i) $\log_5 3125 = 5 \iff \square = 5^{\square}$

(ii) $\log_7 1 = 0 \iff 1 = 7^{\square}$

(iii) $\lg \square = -2 \iff \square = 10^{-2}$

(iv) $\log_3 81 = 4 \iff 81 = \square^4$

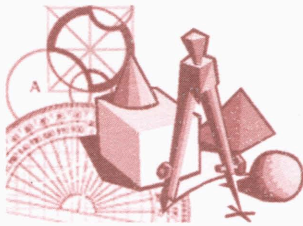
(v) $\log_6 6 = \square \iff 6 = 6^{\square}$

(vi) $\log_{\square} 0.001 = -3 \iff 0.001 = \square^{-3}$

6. $\log_{\square} \square = 3$. Write three suitable pairs of numbers for x and y

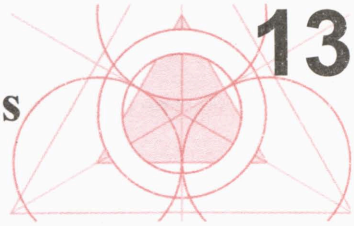
7. $\log_{\square} 64 = \square$. Write four suitable pairs of numbers for x and y

8. $\log_2 \square = \square$. Write three suitable pairs of numbers for x and y



Constructions

13



By studying this chapter you will be able to achieve the following competencies.

- ★ Constructing the four basic loci with accuracy.
- ★ Constructing a perpendicular to a straight line from a point outside the straight line.
- ★ Constructing angles 60° , 90° and angles of multiples of the same.
- ★ Copying an angle equal to a given angle.

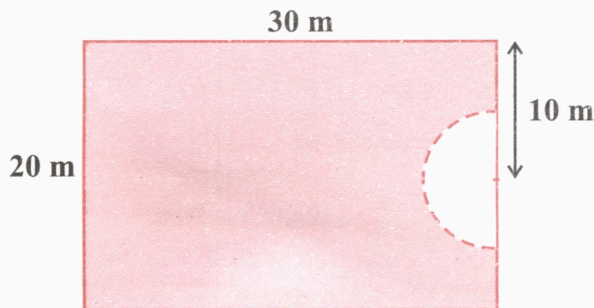
13.1 Loci

If one learns that the locus of a point is the path traced by the point moving according to a given geometrical condition, he will be able to understand many facts about loci.

Let us recollect what we have learnt about loci using the following example.

Example 1

There is a wall around a rectangular green of length 30 m and breadth 20 m. A calf is tied to a point 10 m away from one corner, along a boundary line of the breadth of the green. The length of the rope to which the calf is tied is 7 m. The shaded area in the diagram is the region in which the calf is unable to graze. The path along which the calf can walk with the rope taut or the locus, takes the shape shown below by the dotted lines.



Exercise 13.1



1. A cyclist rides on a bicycle along a straight road on flat land. What is the locus of a point on the seat of the bicycle?
2. Draw the locus of a lower end point of a door of a house, when it is in the process of opening.

13.2 Basic Loci

Let us identify the four basic loci and their geometrical constructions. You who are preparing for it, should have a pencil with a sharp point. Also prepare with,

- ★ a pair of compasses.
- ★ a smooth straight edge.
- ★ two set squares with smooth edges.

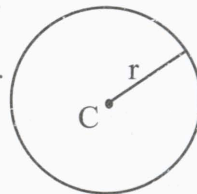
1. The locus of a point moving at a constant distance from a given fixed point.

The locus of a point moving at a constant distance from a fixed point is a circle.

Construction

- ★ Mark a fixed point (let it be C)
- ★ Set the pair of compasses such that the distance between the metal point and pencil point is equal to the constant distance (let this be r)
- ★ Draw a circle keeping the metal point on the fixed point.

The construction will be equal in shape to the diagram shown here. This circle is the locus of a point moving at a constant distance from the fixed point



2. The locus of a point equidistant from two given points.

The locus of a point equidistant from two given points is the perpendicular bisector of the straight line segment which joins the two given points'

Construction

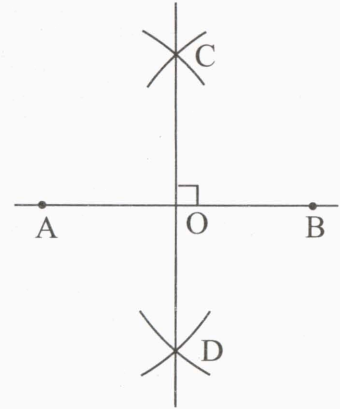
- ★ Mark the two given points. (let them be A and B)
- ★ Draw the line segment which joins the two points. (join AB).



- ★ Set the pair of compasses such that the radius is equal to more than $\frac{1}{2}$ of the length of the line segment drawn. Then taking A as centre draw two arcs on either side of the line segment.
- ★ Also taking B as centre draw two arcs with the same radius on either side of the line segment.
- ★ Name the two points of intersection of the arcs as C and D.
- ★ Join CD. It will be the perpendicular bisector of the line segment AB.



If the perpendicular bisector of the straight line AB meets the straight line AB at O; using the protractor measure the angles \hat{COB} , \hat{AOC} , \hat{AOD} and \hat{DOB} and check whether each angle is equal to 90° . Also measure and check whether $AO = OB$.



The perpendicular bisector of the straight line which joins two given points is the locus of the point equidistant from the two given points.

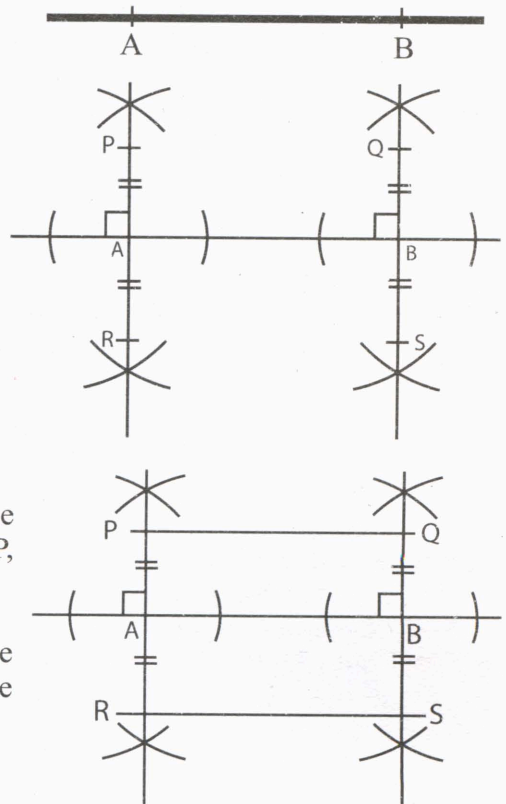
3. The locus of a point equidistant from a given straight line.

The locus of a point equidistant from a given straight line is a straight line parallel to the given straight line.

Construction

- ★ Draw the given straight line
- ★ Mark two points A and B on the straight line.
- ★ Construct two perpendiculars to the given straight line, at the points A and B

- ★ Mark two points on both sides of each perpendicular at equal distance from the given straight line. Name the points as P, R, and Q, S
- ★ Join PQ and RS
- ★ The pair of straight lines PQ and RS are the loci of the points equidistant from the given straight line.

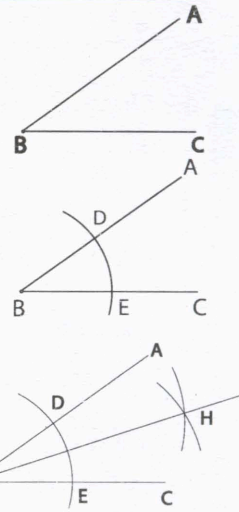


4. The locus of a point equidistant from two intersecting straight lines.

The locus of a point equidistant from two intersecting straight lines is the bisector of the angle formed by the two intersecting straight lines

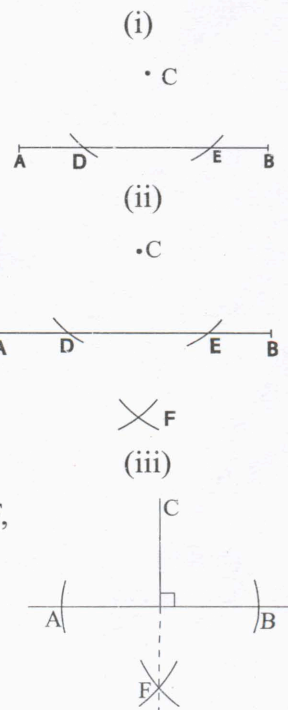
Construction

- ★ Draw two intersecting straight lines. Assume the straight lines to be AB and BC which intersect at B.
- ★ With B as centre and with radius of a length less than BA or BC draw an arc to cut the line segments AB and BC at D and E respectively.
- ★ With the same radius or a suitable radius and with D as centre draw an arc of a circle. Also with the same radius and with E as centre draw an arc such that the two arcs intersect at H.
- ★ Join B and H.
- ★ BH is the bisector of $\angle ABC$. BH is also the locus of the points equidistant from AB and BC



13.3 Constructing a perpendicular to a straight line from an external point.

- ★ Name the given straight line segment as AB.
- ★ Mark the external point as C.
- ★ Taking C as the centre, draw an arc of a circle to intersect the given straight line at two points. Name these intersecting points as D and E.
- ★ Taking the same radius or a suitable radius and D and E as centres, draw two arcs to intersect each other, on the side opposite the side on which the given point is placed. Name the point of intersection as F.
- ★ Placing the straight edge along the points C and F, draw a straight line segment from C to AB.



This is the perpendicular drawn from the external point to the given straight line. CF is the perpendicular drawn from the point C to the line AB. Measure the angles and confirm that CF is perpendicular to AB.

Exercise 13.2

1. Draw a line of length 7 cm. Using the pair of compasses divide it into two equal parts. Measure and see whether the length of one part is 3.5 cm, and check the accuracy of the construction.
2. Draw a straight line segment of length 5 cm. Construct the locus of the points 2 cm away from it.
3. Construct the locus of a point moving 3.8 cm away from a given point C. Describe the locus obtained.
4. Construct the locus of a point moving 4 cm away from a given point A. Mark a point B on the locus, Construct also the locus of a point moving 4 cm away from B. Shade the common region bounded by the two loci. What is the special feature that you observe in these loci?
5. Construct the locus of a point moving 2.5 cm away from a straight line AB.
6. Using the protractor draw the following angles in your exercise book and construct their bisectors.
 - (i) 30°
 - (ii) 65°
 - (iii) 118°
 - (iv) 250°
7. A and B are two points placed 8 cm apart. A point P moves such that $AP \leq BP$. Shade the possible region in which P can exist.
8. AB is a straight line of length 6 cm. A point C is placed 5 cm away from A and 6 cm away from B. Find the position of C and construct a perpendicular to AB from C.
9. Using the knowledge of constructing perpendicular bisectors, divide a straight line of length 19.5 cm into 4 equal parts.

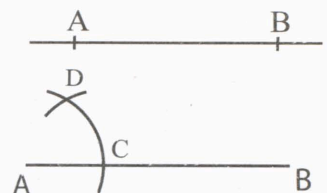
13.4 Constructing Angles

Though angles can be drawn using the protractor or the two set squares, what is expected here is to construct angles using the pair of compasses and a straight edge. Let us construct some angles using these instruments.

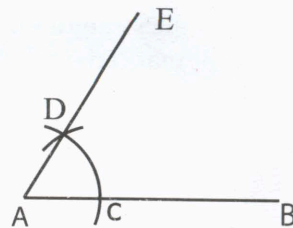
13.5 Construction of an angle of 60°

Steps of the construction

- ★ Draw a straight line segment and name it as AB.
- ★ Taking A as the centre and with a radius less than the length of AB, draw an arc. Let C be the point at which the arc meets AB.



- ★ Taking C as a centre and with the same radius draw an arc to intersect the earlier arc at D.
- ★ Draw a straight line through A and D.

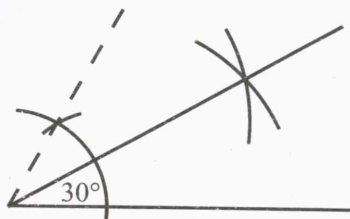


- ★ Name it as AE.
- ★ Then $\hat{BAE} = 60^\circ$

The accuracy of the construction will be higher if the arc is about 3cm.

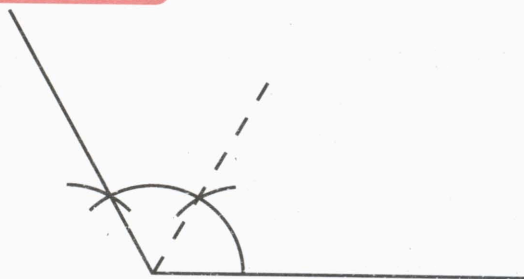
13.6 Construction of an angle of 30°

- First construct an angle of 60°
- Bisect that angle. The resulting angle will be an angle of 30°



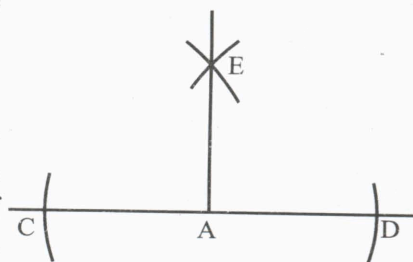
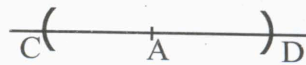
13.7 Construction of an angle of 120°

- ★ Construct an angle of 60° , as before.
- ★ Construct another angle of 60° from there.
- ★ The result will be an angle of 120°



13.8 Construction of an angle of 90°

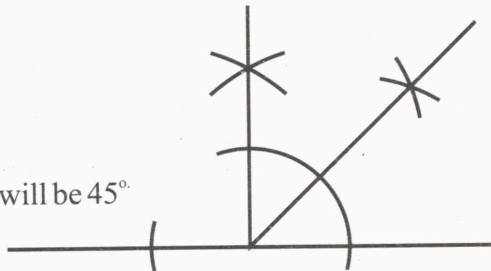
- ★ Mark a point on a straight line, where the angle of 90° is to be constructed.
- ★ Take a radius of suitable length. (about 2 cm - 3 cm)
- ★ With the point at which the angle of 90° is to be constructed as centre, draw two arcs on either side to intersect the straight line. (C and D)
- ★ Taking the radius a little greater than the earlier radius (about 3 cm - 4 cm), draw two arcs to intersect each other with the centres as the points of intersection of the straight line and the arcs drawn earlier.



- ★ Draw the straight line joining the point of intersection of the arcs (E) and the point marked on the straight line A. This line is perpendicular to the given line. That is, the angle between the two lines is 90° .

13.9 Construction of an angle of 45°

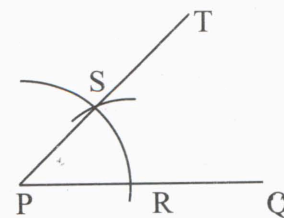
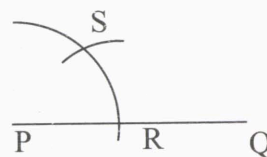
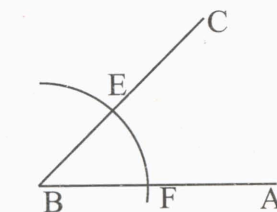
- ★ Construct an angle of 90° as before.
- ★ Bisect that angle. The resulting angle will be 45° .



13.10 Copying an angle

There should be a diagram of the given angle here.

- ★ Name the given angle as $\hat{A}BC$
- ★ Draw a straight line segment PQ
- ★ With B as the centre and the radius less than the length of the arms of $\hat{A}BC$, draw an arc to cut the arms AB and BC at F and E respectively.
- ★ With the same radius and with P as centre draw an arc to cut PQ at R.
- ★ Taking the length EF as the radius and with R as the center, draw an arc to cut the arc drawn earlier at S.
- ★ Draw a straight line joining PS. Name that line as PT. Angles $\hat{A}BC$ and $\hat{Q}PT$ are equal.



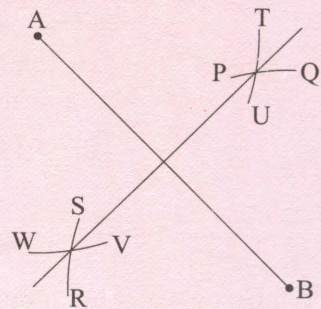
(Measure the angles with the protractor and check whether they are equal)

In this construction it is important to keep the length between the points of intersection (EF) of the arms of the given angle as a radius without changing. Also, it is possible to obtain angles of magnitude twice or three times the given angle, by producing arcs along the earlier arc more than once and marking EF repeatedly.

Exercise 13.3

- Using only the bisection of angles and the copying of angles, construct the following angles
 (i) 15° (ii) 75° (iii) 90° (iv) 120°
- Construct the following angles.
 (i) 30° (ii) 15° (iii) 22.5° (iv) 75° [Hint $30^\circ + 45^\circ$]
 Use the protractor and check the accuracy of the angles constructed.

- Using only the pair of compasses, the straight edge and a pencil, construct a square with 6 cm as the length of a side.
- The given diagram shows how a bisector of a straight line is constructed. The steps of the process is given below. Fill in the blanks by selecting the appropriate word which is given in symbols, letters or numbers within the brackets.

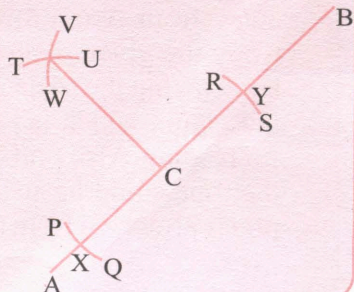


Step 1 Draw the given straight line AB.

Step 2 Taking -----
 (one half of AB/ more than half the length of AB/ less than half the length of AB) as radius and with B as centre draw the arcs PQ and RS.

Step 3 Taking on the pair of compasses the ----- (radius taken in step 2/ radius greater than the one in step 2/ radius smaller than the one in step 2) and with ----- (A/B) as centre draw the arcs TU and WV

Step 4 By joining the points of intersection of the arcs drawn in the above steps obtain the perpendicular passing through the mid point of AB.



- The given diagram shows the process of constructing a perpendicular to a straight line AB at a point C on the straight line.

Fill in the blanks by selecting the suitable term which is given within brackets.

- Step 1** Draw the given line AB and mark the point 'C' on it.
- Step 2** Taking the point ----- (A/B/C) as centre and with a suitable radius mark the points X and Y on AB.
- Step 3** Taking the point ----- (A / B / C / X / Y) as centre and more than half of ----- (AX/XC/ CY/XY/YB) as radius draw the arc TU.
- Step 4** Taking the point -----(A / B / C / X / Y) as centre and with ----- (the same length as that of step (3)/ a length greater than that of step (3)/ a length less than that of step (3) as radius) draw the arc VW.
- Step 5** By joining the point of intersection of the arcs and the point ----- (A/B/C/X/Y) obtain the perpendicular to AB at the given point C.

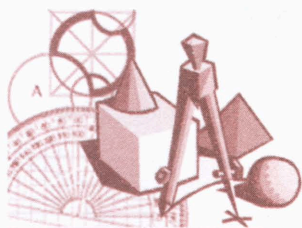
Miscellaneous Exercises

- Using a pair of compasses and a straight edge, construct a rectangle ABCD in which $AB = 8$ cm, $BC = 12$ cm. Find the location of a point E which is 5 cm away from AB and 8 cm away from D.

Measure the distance to EC from each location A, B, C, D.

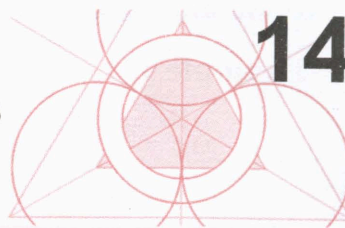
- ABCD is a rectangular land in which $AB = 8$ m and $BC = 16$ m. A calf is tied to the point A with a rope of length 9.5 m. The perpendicular bisector of AC divides the land into two parts. The part which is away from A is grown with vegetables. Copy the given diagram in your exercise book taking the scale as 1 cm to 1 m. Shade the region which is accessible to the calf for feeding. Also find the maximum length the rope could be such that the vegetable plot is protected from the calf.





Equations

14



By studying this chapter you will be able to achieve the following competencies.

- ★ Solving simple equations with two different types of brackets.
- ★ Solving simple equations involving fractions.
- ★ Solving simultaneous equations with one variable having the same coefficient.

My mother has Rs25 more than the amount of money I have

My father has three times the amount of money I have

My son has Rs25 less than the amount of money I have

My son has one third of the amount of money I have

Rs75

Rs50

Rs150

14.1 Solving simple equations

When solving equations, it is essential to know how an equation is formed. Consider how the following equations are formed.

★ $\frac{(2x - 3)}{5} = 1$

When 3 is subtracted from twice the number 'x' and the result is divided by 5, the answer is 1

★ $4\left(\frac{a}{2} + 3\right) = 8$

When 3 is added to half the number 'a' and the result is multiplied by 4, the answer is 8.

★ $\frac{(5 - 3y)}{2} + 3 = 4$

When the number 'y' is multiplied by -3, and added to 5, and the result is divided by 2, and 3 is added to this result, then the answer is 4.

Now we will see how the equations given below are solved.

Example 1

$$\frac{(2x-3)}{5} = 1$$

$$\frac{(2x-3)}{5} \times 5 = 1 \times 5 \quad (\text{multiply both sides of the equation by } 5)$$

$$2x - 3 = 5$$

$$2x - 3 + 3 = 5 + 3 \quad (\text{add } 3 \text{ to both sides})$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

(divide both sides of the equation by 2)

$$\underline{\underline{x = 4}}$$

Example 2

$$4\left(\frac{a}{2} + 3\right) = 8$$

$$\frac{4\left(\frac{a}{2} + 3\right)}{4} = \frac{8}{4}$$

$$\frac{a}{2} + 3 = 2$$

$$\frac{a}{2} + 3 - 3 = 2 - 3$$

$$\frac{a}{2} = -1$$

$$\frac{a}{2} \times 2 = -1 \times 2$$

$$\underline{\underline{a = -2}}$$

Example 3

$$\frac{(5-3y)}{2} + 3 = 4$$

$$\frac{(5-3y)}{2} + 3 - 3 = 4 - 3$$

$$\frac{(5-3y)}{2} = 1$$

$$\frac{(5-3y)}{2} \times 2 = 1 \times 2$$

$$5 - 3y = 2$$

$$5 - 3y - 5 = 2 - 5$$

$$-3y = -3$$

$$\frac{-3y}{-3} = \frac{-3}{-3}$$

$$\underline{\underline{y = 1}}$$

Exercise 14.1

1. Explain in words, how the following equations are formed.

(i) $\frac{x}{2} - 3 = 5$

(ii) $3 + 2a = -1$

(iii) $\left(\frac{x}{2} + 1\right) = 10$

$$(iv) 5\left(\frac{3x}{2} - 1\right) = 5 \quad (v) \frac{3p-1}{4} = 2$$

2. Solve the equations given below.

$$(i) 5x - 2 = 8$$

$$(ii) 3x - 4 = -10$$

$$(iii) 2x - 5 = x + 1$$

$$(iv) \frac{2-x}{5} = 4$$

$$(v) 5(a-3) - 2 = 8$$

$$(vi) 3(x-1) = 2(x-4)$$

$$(vii) 5 - \frac{x}{2} = -3$$

$$(viii) \frac{3x}{2} = x + 6$$

$$(ix) \frac{a}{2} - \frac{a}{3} = 5$$

$$(x) \frac{1}{3}\left(\frac{2x}{3} - 3\right) = -1$$

3. (i) Nimal bought 8 pens and paid the shop keeper with a Rs100 note. He received a balance of Rs.4. Form an equation using Rs x as the price of one pen. Hence find the price of a pen.
- (ii) The amount of money my brother has is Rs.20 more than twice the amount I have. The total amount of money both of us have is Rs.110.
- (a) If I have Rs. a , find an expression in terms of a for the amount of money my brother has.
- (b) Form an equation, and find separately the amount of money each one has.
4. The length of a rectangle is 5 cm longer than twice its breadth. Its perimeter is 52cm. Find the length and breadth of the rectangle.
5. The length of a side of a certain equilateral triangle is twice the length of a side of a certain square. The perimeter of the equilateral triangle is 30 cm more than the perimeter of the square. Find the length of a side of the square and the length of a side of the equilateral triangle.

14.2 Solving equations with two types of brackets.

Using brackets

Types of brackets

()

{ }

[]

Parenthesis

**Braces
(Curly brackets)**

Square brackets

(We have already learnt how to use parenthesis.)

Brackets should be used in the following order.

[{ () }]

Removing brackets

Remove brackets starting from the inner most bracket and moving gradually to the outermost brackets as follows.

Parenthesis \longrightarrow Braces \longrightarrow Square brackets etc,

Example 4

Remove brackets and solve the equations.

$$5\{3(x+2)+2\}=10$$

$$5\{3(x+2)+2\}=10$$

$$5\{3x+6+2\}=10 \quad (\text{Remove the Parenthesis first})$$

$$5\{3x+8\}=10$$

$$15x+40=10 \quad (\text{Then remove the braces})$$

$$15x+40-40=10-40$$

$$15x=-30$$

$$\frac{15x}{15}=\frac{-30}{15}$$

$$\underline{\underline{x=-2}}$$



Exercise 14.2



Solve

(i) $2\{2(5-x)+3\}=-2$

(ii) $3\{3(x+2)-2(x-1)\}=0$

(iii) $5+2\{x-3(1-x)\}=7$

(iv) $4-3\left\{\frac{1}{2}(2x-4)+3x+2\right\}=0$

(v) $2\left\{2\left(\frac{x}{2}-1\right)+3\right\}=6$

14.3 Solving simultaneous equations

Consider the pair of linear equations with two variables, given below.

$$x+y=5$$

If x and y are integers, consider a few pairs of values which satisfy the equation

$$x-y=1$$

If x and y are integers, consider a few pairs of values which satisfy the equation

x	y
'	'
'	'
-1	+6
0	5
1	4
2	3
3	2
4	1
'	'
'	'

There are an infinite number of pairs of values which satisfy the equation $x+y=5$

x	y
'	'
'	'
6	5
5	4
4	3
3	2
2	1
1	0
'	'
'	'

There are an infinite number of pairs of values which satisfy the equation $x-y=1$

But there is only one pair of values which satisfies both equation $x+y=5$ and $x-y=1$. That is $x=3$ and $y=2$. This is the solution of the pair of equations given above. A pair of equations with two variables such as the above is called a pair of simultaneous equations.

Let us now see how a pair of simultaneous equations is solved.

Example 5

Solve the simultaneous equations.

(i) $a+b=2$
 $a-b=-4$

First name them, for the convenience of reference

$$a+b=2 \quad \text{---(1)}$$

$$a-b=-4 \quad \text{---(2)}$$

Method 1

We can solve the equations by removing one of the variables in the above equations. To do this, the coefficient of the variable which we want to remove must be equal in both equations. By adding the above equations, 'b' can be removed while by subtracting the equations, 'a' can be removed.

(1)+(2)

$$a+b+a-b=2-4$$

$$2a=-2$$

$$\frac{2a}{2} = \frac{-2}{2}$$

$$\underline{\underline{a=-1}}$$

The value of 'b' can be found by substituting the value of 'a' in either of the equations.

Substitute the value of a in (1)

$$\begin{aligned} a+b &= 2 \\ -1+b &= 2 \\ -\cancel{x}+b+\cancel{x} &= 2+1 & a &= -1 \\ \underline{\underline{b}} &= 3 & b &= 3 \end{aligned}$$

Method 2 (by comparison)

Here the same variable is made the subject of each equation.

From (1)

$$\begin{aligned} a+b &= 2 \\ a+b-b &= 2-b \\ a &= 2-b \text{ ————— (3)} \end{aligned}$$

From (2)

$$\begin{aligned} a-b &= -4 \\ a-\cancel{b}+\cancel{b} &= -4+b \\ a &= -4+b \text{ ————— (4)} \end{aligned}$$

By equating the expressions in (3) and (4)

$$\begin{aligned} 2-b &= -4+b \\ 2-b+4 &= -4+b+4 \\ 6-b &= b \\ 6-\cancel{b}+\cancel{b} &= b+b \\ 6 &= 2b \\ \frac{6}{2} &= \frac{2b}{2} \\ \underline{\underline{b}} &= 3 \end{aligned}$$

Substitute the value of b in (3) [It can also be substituted in (4)]

$$\begin{aligned} a &= 2-b \\ a &= 2-3 & a &= -1 \\ \underline{\underline{a}} &= -1 & b &= 3 \end{aligned}$$

Example 6

Method 1

$$3x+y=5 \text{ ————— (1)}$$

$$x+y=-3 \text{ ————— (2)}$$

$$(1)-(2)$$

$$3x+y-(x+y)=5-(-3)$$

$$3x+y-x-y=5+3$$

$$2x=8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$\underline{\underline{x}} = 4$$

Substitute the value of x in (2)

$$x + y = -3$$

$$4 + y = -3$$

$$4 + y - 4 = -3 - 4$$

$$\underline{\underline{y = -7}}$$

$$x = 4$$

$$y = -7$$

Method II

$$3x + y = 5 \text{ ————— (1)}$$

$$\text{From (1) } y = 5 - 3x \text{ ————— (3)}$$

$$x + y = -3 \text{ ————— (2)}$$

$$\text{From (2) } y = -3 - x \text{ ————— (4)}$$

By equating the expressions in (3) and (4)

By Substituting the value of x in (3)

$$5 - 3x = -3 - x$$

$$5 + 3 = -x + 3x$$

$$8 = 2x$$

$$\underline{\underline{x = 4}}$$

$$y = 5 - 3x$$

$$y = 5 - 3 \times 4$$

$$y = 5 - 12$$

$$\underline{\underline{y = -7}}$$

$$x = 4$$

$$y = -7$$

Exercise 14.3

Solve the pairs of simultaneous equations given below.

1. $a + b = 7$

$$a - b = 3$$

2. $2x - y = 7$

$$3x + y = 8$$

3. $2a - b = 10$

$$a + b = -1$$

4. $3x + y = 7$

$$x + y = 1$$

5. $x - 2y = -1$

$$x - 5y = -7$$

6. $p = 2q + 3$

$$p + q = 9$$

7. $7a - 3b = 5$

$$a + 3b = 3$$

8. $3c - 2d = 5$

$$3c + d = -1$$

9. $3m - 2n = -5$

$$n - 3m = 1$$

10. $\frac{x}{2} - y = 3$

$$\frac{x}{2} + y = 5$$

11. $\frac{2x}{3} - y = 1$

$$3y - \frac{2x}{3} = 1$$

12. $\frac{a}{2} + b = 4$

$$\frac{a}{2} - 2b = 1$$

Lesson Sequence, Competency Levels and Number of Periods

Content	Competency Levels	Number of Periods
1st Term		
01 Rounding Off and Scientific Notation	1.1, 1.2	05
02 Number Patterns	2.1	05
03 Fractions	3.1	06
04 Percentages	5.1	05
05 Simple Interest	5.2	06
06 Algebraic Expressions	14.1, 14.2	06
07 Factors of Algebraic Expressions	15.1	06
08 Angles Related to Straight lines and Parallel Lines	21.1, 21.2	07
09 Liquid Measures	11.1	05
		51
2st Term		
10 Direct Proportions	4.1	04
11 Calculator	6.3	04
12 Indices and Logarithms	6.1, 6.2	08
13 Constructions	27.1, 27.2	05
14 Equations	17.1, 17.2	08
15 Angles of a Triangle	23.1, 23.2	06
16 Formulae	19.1	05
17 Circumference of a Circle	7.1	05
18 Pythagoras Relationship	23.4	07
19 Graphs	20.1	05
		59
3st Term		
20 Inequalities	20.2	04
21 Sets	30.1	05
22 Area	8.1, 8.2	08
23 Probability	31.1	05
24 Angles of Polygons	23.3	06
25 Algebraic Fractions	16.1	05
26 Volume	10.1	05
27 Scale Drawings	13.1, 13.2	05
28 Representation of Data and Interpretation of Data	28.1, 29.1	08
		51

தமிழ் அரிச்சுவடி

∴	அ	ஆ	இ	ஈ	உ	ஊ	எ	ஏ	ஐ	ஓ	ஔ	ஐள
க்	க	கா	கி	கீ	கு	கூ	கெ	கே	கை	கொ	கோ	கௌ
ங்	ங	ஙா	ஙி	ஙீ	ஙு	ஙூ	ஙெ	ஙே	ஙை	ஙொ	ஙோ	ஙௌ
ச்	ச	சா	சி	சீ	சு	சூ	செ	சே	சை	சொ	சோ	சௌ
ஞ்	ஞ	ஞா	ஞி	ஞீ	ஞு	ஞூ	ஞெ	ஞே	ஞை	ஞொ	ஞோ	ஞௌ
ட்	ட	டா	டி	டீ	டு	டூ	டெ	டே	டை	டொ	டோ	டௌ
ண்	ண	ணா	ணி	ணீ	ணு	ணூ	ணெ	ணே	ணை	ணொ	ணோ	ணௌ
த்	த	தா	தி	தீ	து	தூ	தெ	தே	தை	தொ	தோ	தௌ
ந்	ந	நா	நி	நீ	நு	நூ	நெ	நே	நை	நொ	நோ	நௌ
ப்	ப	பா	பி	பீ	பு	பூ	பெ	பே	பை	பொ	போ	பௌ
ம்	ம	மா	மி	மீ	மு	மூ	மெ	மே	மை	மொ	மோ	மௌ
ய்	ய	யா	யி	யீ	யு	யூ	யெ	யே	யை	யொ	யோ	யௌ
ர்	ர	ரா	ரி	ரீ	ரு	ரூ	ரெ	ரே	ரை	ரொ	ரோ	ரௌ
ல்	ல	லா	லி	லீ	லு	லூ	லெ	லே	லை	லொ	லோ	லௌ
வ்	வ	வா	வி	வீ	வு	வூ	வெ	வே	வை	வொ	வோ	வௌ
ழ்	ழ	ழா	ழி	ழீ	ழு	ழூ	ழெ	ழே	ழை	ழொ	ழோ	ழௌ
ள்	ள	ளா	ளி	ளீ	ளு	ளூ	ளெ	ளே	ளை	ளொ	ளோ	ளௌ
ற்	ற	றா	றி	றீ	று	றூ	றெ	றே	றை	றொ	றோ	றௌ
ன்	ன	னா	னி	னீ	னு	னூ	னெ	னே	னை	னொ	னோ	னௌ

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Name of the school.....

Year	Name of student who is using the book	Class	Signature of the class teacher
2011
2012
2013
2014

