THE OPEN UNIVERSITY
MAF 1604
MATHEMATICS FOR SCIENCE AND TECHNOLOGY

STATICS AND HYDRO–STATICS
CO-PLANAR FORCES

Block 9

Unit 2

OPEN UNIVERSITY OF SRI LANKA

FOUNDATION COURSE

FOR

SCIENCE AND TECHNOLOGY

MAF 1604 - MATHEMATICS

BLOCK 9 - STATICS AND HYDRO-STATICS

UNIT 2 - CO-PLANAR FORCES

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INTRODUCTION

Statics deals with the study of forces in equilibrium. Conditions that must be satisfied by a system of forces acting on a partice or a system of rigid bodies such that they be in equilibrium are obtained for different conditions. This block on statics consists of seven units as follows :

Unit 1 - Vectors

Unit 2 - Coplaner forces

Unit 3 - System of forces

Unit 4 - Centre of gravity

Unit 5 - Frameworks

Unit 6 - Statics of fluids

Unit 7 - Virtual work

2. **STATICS**

2.1. Introduction

Statics is the science which deals with the study of bodies or particles which are at rest.

A particle is a part of matter which is so small that the position may be given as that of a point. A body could be considered as formed of an infinitely large number of particles.

Let us begin by discussing the concept of force.

2.2. Force

You may have observed that a body which is at rest can be moved by exerting a pull or push. Also, the direction of a moving body can be changed by exerting a push or a pull. Such a pull or a push is an example for a force.

So a force would change or would tend to change the state of rest or uniform motion of a body.

2.3. Important Properties of Forces

Let us consider a body which is initially at rest. To make this object move a certain force has to be applied. We know by experience that we need a large force to move a heavy object and a smaller force to move a lighter object. Hence a force is a physical quantity which has a certain magnitude. Also, the direction of motion would depend on the sense in which it is applied. So, a force has a direction and this is the second property of force. The third property of force is the line of action. Consider the forces applied to the object shown in Fig. 2.1(a).

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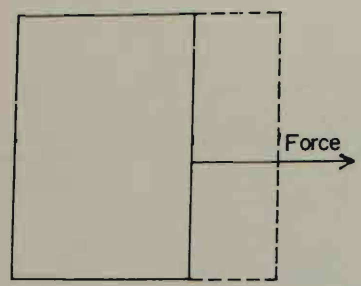

Fig. 2.1(a)

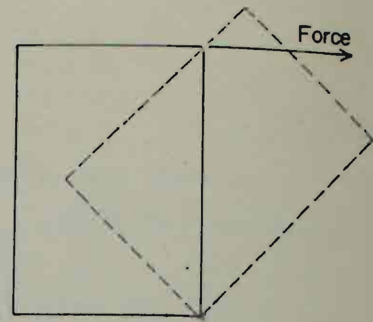

Fig. 2.1(b)

The force is applied at a middle point of the object and will move it to a position shown by the dotted lines. In Fig. 2.1(b), the same force (same in direction and magnitude) is applied at the top of the object. In this instant, the force can topple the object as shown by the dotted lines. So you can see that the force with same direction and magnitude applied to the same object at two different points can produce entirely different results.

Hence, a force will be completely known, when we know

(i) its magnitude,

(ii) its direction

(iii) its line of action.

2.4. Equal Forces

We say two forces are equivalent when they have the same magnitude, direction and point of application.

2.5. Units of Forces

Some units for measuring force are Newton, Dyne and Poundal. You will at a later stage learn as to how these units are derived.

2.6. Representation of a Force

Since force has both a magnitude and a direction we can conveniently represent it by an arrow (or a directed line segment) drawn through its point of application. The direction of the arrow represents the direction of the force while the length of the arrow would be proportional to the magnitude of the force.

Example 2.1

A certain force has a magnitude of 20 newtons (abbreviated as N) and is pointed in the direction of north. Represent this force by a directed line segment.

The first step should be to select a suitable scale to represent the magnitude of the force. Let us assume that a length of 1 cm represents a force of 5N in magnitude.

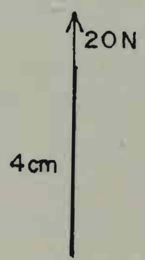

Fig. 2.2.

Hence the length of the line segment should be 4 cm. Its direction is given to be northwards.

Therefore, a line segment with a length of 4 cm and pointed in the direction of north will represent the given force in magnitude and direction.

Notation

Forces will be denoted in this text by drawing a line under the letter e.g. F, P. The magnitude of the force will be denoted by the letter only. e.g. F, P. It is important to understand the meanings of two words mass and weight. The mass m of a body is the quantity of matter in the body. The idea of weight is one with which everyone is familiar. The earth attracts a body to itself with a force which is called the weight of the body. We can write $W = mg$ where g is the gravitational force on a body of unit mass.

2.7. Subdivisions of Force

There are three different forms of a force.

(i) attraction

(ii) tension

(iii) reaction

(i) Attraction

An attraction is a force exerted by one body on another without the intervention of any visible instrument and without the bodies being necessarily in contact. The most common example is the Gravitational attraction which the earth has on a body.

(ii) Tension

When a string is used to suspend a weight or move a body the string is in a state of tension. If we consider a string of negligible weight supporting a weight W vertically the tension in the string is approximately the same through its length and may be taken to be equal to W.

If however, the string is heavy the tension in the string varies from point to point. If the weight of the string can be neglected we refer to it as a light string.

We note that the tension in a light string passing over a smooth edge is the same throughout its length. However, if two or more strings be knotted together the tensions need not necessarily be the same in each string.

(iii) Reaction

If a body leans, or is pressed against another body, each body experiences a force at the point of contact. Such a force is called a Reaction.

2.8. Smooth Bodies

When a body presses against another, there is a force acting along their common surface. This force tends to prevent slipping. This is due to the friction between the surface and is known as the frictional force. With highly polished surfaces, this force may be very small. If this force is zero, the surfaces are said to be perfectly smooth. In such cases, the only force between the bodies acts perpendicularly to their common surface. This force is called the normal reaction between the two bodies.

For example, when a rod rests with one end against a smooth plane, the reaction at that end is normal to the plane.

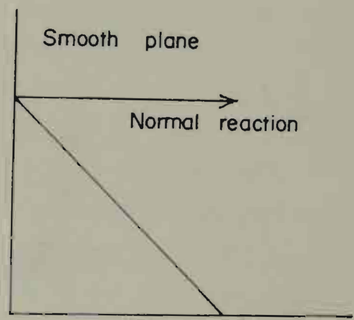

Fig. 2 .3.

So far we have spoken of only a single force acting on a body or a particle. In physical situations, however, one comes across a number of forces rather than a single force, acting on a body. Hence we shall next consider a system of forces.

2.9. A System of Forces

A set of forces acting simultaneously on a body is called a system of forces.

For example, consider the case of a block of wood resting on a

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smooth inclined plane, supported by a force P. The system of forces on the block of wood consists of

(i) its weight W

(ii) reaction R of the plane on the block and

(iii) the applied force P.

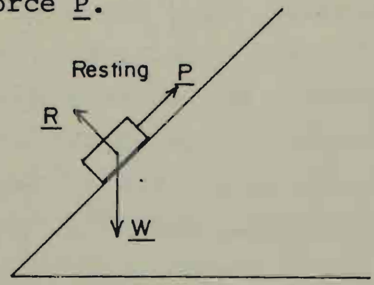


Fig. 2.4.

2.10. Co-planar Forces

A system of forces is said to be co-planar, if the lines of action of all the forces lie in the same plane.

2.11. The Unit of a Force

A force is measured in terms of the force of gravity on a standard mass. In the metric system, the standard force is the force with which the earth attracts a mass of one Kilogramme. This is called one Kilogramme weight, and written as 1 Kgf. In the British system, the standard force is one Pound weight.

However, since these are all gravitational forces, their magnitudes would not be the same at all localities. Hence it is necessary to define an absolute measure of a force.

The unit of the absolute measure of force in the MKS System is the Newton (written as N). This unit of force is defined as that force which would give to a one Kilogramme mass an acceleration of one metre per second per second.

Thus 1 Kgf = g N where g is acceleration due to gravity. g varies from place to place and is approximately 9.81 metres per second per second.

2.12. Composition and Resolution of Coplanar Forces

Composition of Coplanar Forces

Let us consider two forces of magnitude P and Q acting on a body as shown in the Figure 2.5.

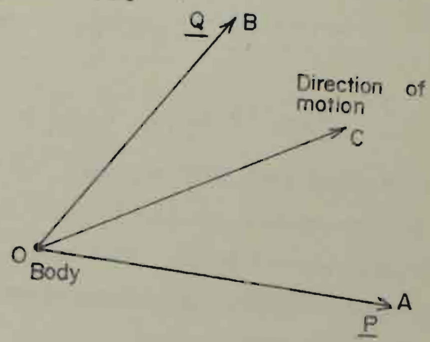


Fig. 2.5.

As an example, if we pull the body in the direction of P and Q by means of strings attached to it you will observe that the body is moving in the direction OC.

The same result may be produced by applying a single force along the direction OC. This force is the resultant of the two forces P and Q. This concept can be extended to the case of a system of forces. The resultant of a system of forces is the single force which would produce the same effect as the system of forces acting on the body.

Next, we shall study the "Parallelogram Law of Forces" which enables one to determine the resultant of two forces acting on a body.

2.13. The Parallelogram Law of Forces

If two forces P and Q, acting on a particle at O, be represented in magnitude and direction by the straight lines OA and OB, then the resultant force $\underline{R}$ is represented in magnitude and direction by the diagonal OC of the parallelogram OACB.

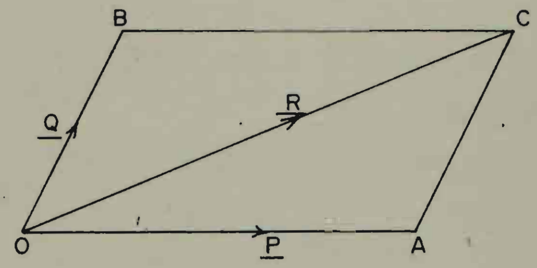


Fig. 2.6.

In the simple case, when the two forces are in the same direction their resultant is clearly equal to their sum as in Fig. 5.7(a). When they act in opposite directions, the magnitude of the resultant is equal to their difference and the direction of the resultant is that of the greater force.

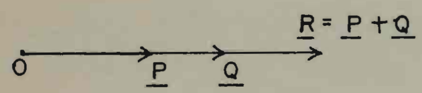


Fig. 2.7(a)

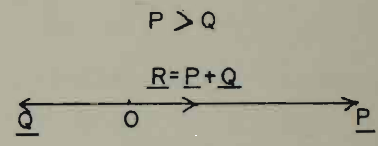


Fig. 2.7(b)

In Fig. .7(b), if $\underline{P} = \underline{Q}$, then $\underline{R} = 0$. On the other hand, if $\underline{R}=0$, $\underline{P} = \underline{Q}$. Thus the resultant force is zero only if the two forces are equal in magnitude and act in opposite directions.

Now, let us determine the megnitude and direction of the resultant of two forces P and Q acting at a point O and inclined at angle to each other.

The resultant force $\underline{R}$, is represented by the diagonal OC of the parallelogram OACB.

Draw CD perpendicular to OA.

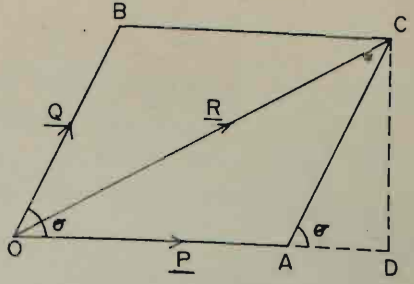


Fig. 2.8

Then by Pythagorus theorem,

$$OC^2 = OD^2 + DC^2$$

$$= (OA + AD)^2 + DC^2$$

$$= (OA + AC\cos\theta)^2 + (AC\sin\theta)^2 \quad \because AD = AC.\cos\theta$$

$$CD = AC.\sin\theta$$

By substituting for OC, OA, AC from R,P,Q respectively, we have,

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$= P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta \quad \text{since } \cos^2\theta + \sin^2\theta = 1$$

Therefore, $R = P^2 + Q^2 + 2PQ\cos\theta$

Hence when P,Q and θ are known R (the magnitude of the resultant) can be obtained from the above formula.

To find the direction of <u>R</u>

$$\tan COD = \frac{CD}{OD} = \frac{AC.\sin\theta}{OA + OD}$$

$$= \frac{AC.\sin\theta}{OA + AC\cos\theta}$$

$$\tan COD = \frac{Q\sin\theta}{P + Q\cos\theta}$$

We observe that, if the forces are at right angles, $\theta = 90^\circ$, so that

$$R = P^2 + Q^2 + 2PQ \cos 90$$

$$= P^2 + Q^2 \quad \text{since } \cos 90 = 0$$

$$\tan COD = \frac{Q \sin 90}{P + Q \cos 90}$$

$$= \frac{Q}{P} \quad \text{since } \sin 90 = 1$$

Example 2.2

Two forces of magnitude 3P and 5P acting on a particle are inclined at an angle of 60°. Find their Resultant.

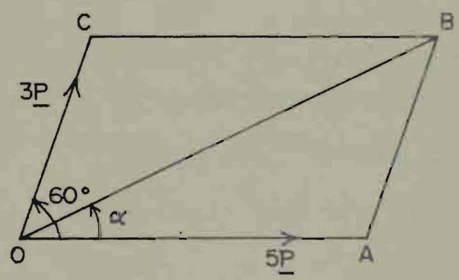


Fig. 2.9

Using the equation

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta ,$$

we get,

$$R^2 = (5P)^2 + (3P)^2 + 2 \times 5P \times 3P \cos 60$$

$$= 25P^2 + 9P^2 + 30P^2 \times \tfrac{1}{2}$$

$$= 49P^2$$

$$\text{Thus } R = 7P$$

Using the equation $\tan BOA = \dfrac{Q \sin \theta}{P + Q \cos\theta}$ we get,

$$\text{i.e. } \tan \alpha = \frac{3P \sin 60}{5P + 3Q \cos 60}$$

$$= \frac{3P \times \frac{\sqrt{3}}{2}}{5P + 3P \times \frac{1}{2}}$$

$$= \frac{3\sqrt{3}}{13} \qquad \therefore \quad \alpha = 21.7867^{\circ}$$

Example 2.3

If the resultant of two forces 7X and 8X is equal to 13X, find the angle between the forces.

In this problem, we are given the two forces P and Q and their resultant R. It is required to find the angle θ between the two forces.

By substituting P = 7X and Q = 8X and R = 13X in the equation,

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta, \text{ we get}$$

$$(13X)^2 = (7X)^2 + (8X)^2 + 2(7X)(8X) \cos \theta$$

$$169X^2 = 49X^2 + 64X^2 + 112X^2 \cos \theta$$

$$56X^2 = 112X^2 \cos \theta$$

$$\cos \theta = \frac{56}{112} = \frac{1}{2}$$

$$\theta = 60^\circ.$$

S.A.Q. 2.1

Two equal weights each equal to W are attached to the ends of a thin string which passes over smooth pegs in a wall, arranged in the form of an equilateral triangle with one side horizontal. Find the thrust on each peg.

2.14. Resolution of a Force

Using the parallelogram law of forces, a given force R acting on a particle may be replaced by two forces. Since we may construct an infinite number of parallelograms on a given line as diagonal, a given force may be resolved into two components in an infinite number of ways.

In practice, the directions of the components are known, and the most important case is when these directions are at right angles to each other.

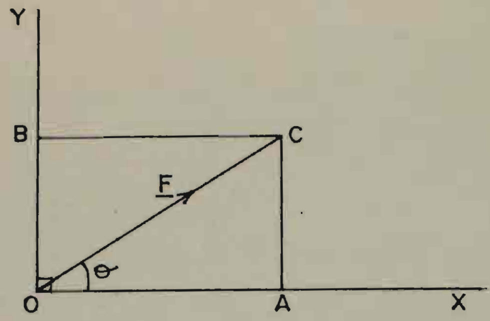


Fig. 2.10

Let OC represent the magnitude of a given force $\underline{F}$, and suppose we wish to resolve it into two components, one along OX, and the other along a direction OY perpendicular to OX.

Draw CA perpendicular to OX and CB perpendicular to OY. Then OA and OB represent the components of $\underline{F}$ along OX and OY respectively.

If the angle COX = θ we have,

$$\cos \theta = \frac{OA}{OC} \text{ and } \sin \theta = \frac{CA}{OC} = \frac{OB}{OC}$$

$$OA = F \cos \theta \text{ and } OB = F \sin \theta \text{ since } OC = F$$

Hence a force $\underline{F}$ is equivalent to a force $\underline{F}$ cos θ along a line making an angle θ with its own direction together with a force $\underline{F}$ sin θ perpendicular to the direction of the first component.

When a force is resolved in this manner into two forces whose directions are at right angles these forces are called the Resolved parts or Resolutes of the given force in these two directions.

The resolved part of a force $\underline{F}$ in a direction making an angle θ with it is $\underline{F}$ cos θ.

<u>Example 2.4</u>

Find the resolutes of the following forces in OX and OY directions.

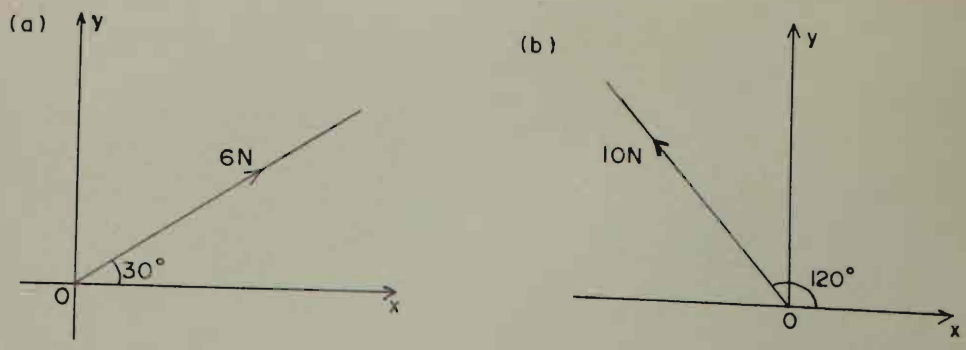


Fig. 2 .11

We shall use an arrow to indicate the direction of resolution of forces.

i.e. → means resolution of forces along the positive x-axis.

(a) $6 \cos 30^{\circ}\ N = 6 \times \frac{\sqrt{3}}{2}\ N = 3\sqrt{3}\ N$

$6 \sin 30^{\circ}\ N = 6 \times \frac{1}{2}\ N = 3N$

(b) $10 \cos 120^{\circ} N = 10 \cos(180^{\circ} - 60^{\circ})\ N$

$= -10 \cos 60^{\circ} N$

$= -10 \times \frac{1}{2}\ N$

$= -5N$

The negative sign is because the assumed positive direction is along the OX direction while the resolute of the force in the x direction acts in the opposite sense.

$10 \sin 60^{\circ}\ N = 10 \cdot \frac{\sqrt{3}}{2}\ N = 5\sqrt{3}\ N$

2.15. Resolution of a Force in two directions making angles α and β with the force

If we do require the components of a force $\underline{F}$ in directions making angles α and β with F they can be found as follows:

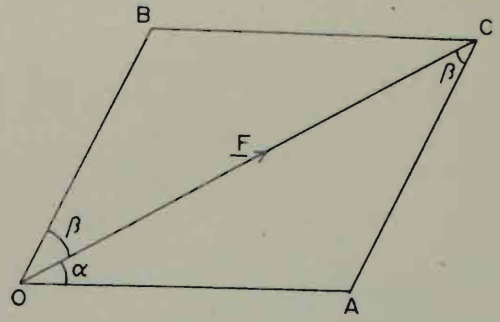


Fig. 2.12

Let OC represent $\underline{F}$. Draw OA and OB, making angles of α and β with OC, and through OC, draw parallels to complete the parallelogram OACB.

Then by the parallelogram law of forces, OA and OB (or OA and AC) represent the required components, in magnitude and direction.

Hence from the triangle OAC,

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin OAC}$$

$$OAC = 180^\circ - (\alpha + \beta)$$

$$\sin OAC = \sin 180^\circ - (\alpha + \beta)$$

$$= \sin(\alpha + \beta)$$

Hence $OA = OC. \dfrac{\sin\beta}{\sin(\alpha + \beta)} = F. \dfrac{\sin\beta}{\sin(\alpha + \beta)}$

and $AC = OC. \dfrac{\sin\alpha}{\sin(\alpha + \beta)} = F. \dfrac{\sin\alpha}{\sin(\alpha + \beta)}$

Notice that the components of a force in the two assigned directions are not the same as the resolved parts of the force in these directions.

For example, the resolved part of $\underline{F}$ in the direction OA is $\underline{F}\cos\alpha$.

Example 2 .5

Resolve a force of 10N in two directions making an angle 30° with the force.

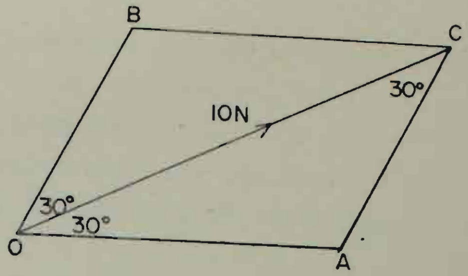


Fig. 2 .13

Let OC represents the given force 10N.

Then the required components are given by

$$OA = 10\frac{\sin 30^\circ}{\sin(30^\circ+30^\circ)}$$

$$\text{and } OB = 10\frac{\sin 30^\circ}{\sin(30^\circ+30^\circ)}$$

$$\text{i.e.}\quad OA = \frac{10 \cdot \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$$

Similarly $OB = \frac{10\sqrt{3}}{3}$

Example 2 .6

Two forces acting on a particle have magnitudes of $4\sqrt{3}$ and 4, Newtons respectively. They are acting at angles of 30° and 120° respectively to the horizontal axis (x-axis) and all the forces are in the same vertical plane. Find the algebraic sum of the resolutes in horizontal and vertical directions.

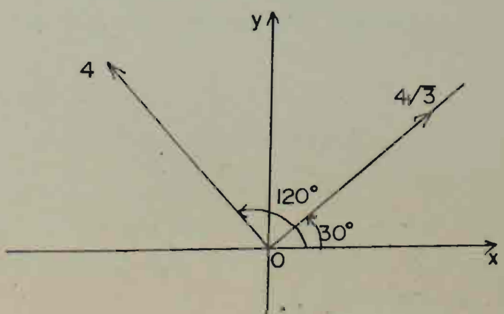


Fig. 2.14

As shown in the Example 5.4, the resolute of a force in a given direction can have a positive or negative sign. To find the algebraic sum of the resolutes of a system of forces in a given direction, the resolutes of the individual forces are calculated and the sum of the resulting algebraic numbers obtained.

Let us first consider the resolution in the OX direction.

Resolute of the $4\sqrt{3}$ N force $= 4\sqrt{3}\cos 30^{o}$N

$= 4\sqrt{3} \times \frac{\sqrt{3}}{2}$ N

$= 6$N

Resolute of the 4N force $= 4\cos 120^{o}$N

$= 4\cos(180^{o}-60^{o})$N

$= -4\cos 60^{o}$N

$= -4 \times \frac{1}{2}$ N $= -2$N

If X is the algebraic sum of the resolutes in the OX direction

$$X = 6-2\text{ N} = 4\text{N}$$

Once you are familiar with the method of calculating, there is no need to show the calculation of the resolute of each and every force separately and the algebraic sum could be obtained directly as shown below:

If Y is the algebraic sum of the resolutes in the OY direction. We have,

$$Y = 4\sqrt{3}\sin 30^{o} + 4\sin 60^{o}\text{ N}$$

$$= 4\sqrt{3} \times \frac{1}{2} + 4 \times \frac{\sqrt{3}}{2}\text{ N}$$

$$= 4\sqrt{3}\text{ N}$$

Example 2.8

Find the resultant of the forces given in Example 1.6 and hence show that the algebraic sum of the resolutes of two forces in OX direction is equal to the resolute of the resultant in the OX direction.

We have shown that, when the forces are at right angles then the resultant R is given by

$$R^2 = P^2 + Q^2$$

Hence

$$R^2 = (4)^2 + (4\sqrt{3})^2$$

$$= 16 + 16 \times 3 = 64$$

Thus $R = 8N$

$$\tan\theta = \frac{4}{4\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

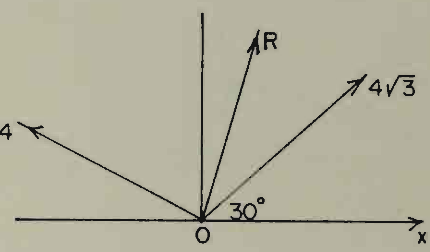


Fig. 2.15

The resolute of the resultant in the OX direction is

$$R\cos(\theta + 30^\circ)N$$

i.e. $8\cos 60^\circ N = 8 \times \frac{1}{2} N = 4N$

This value is equal to the X value which we have calculated in Example 1.6.

Hence the algebraic sum of the resolutes of two forces in OX direction is equal to the resolute of the resultant in the OX direction.

Although we have obtained the above result by considering only the OX direction, the result is true for any direction as stated in the following theorem.

Theorem 2.1

The algebraic sum of the resolutes of two forces in a given direction is equal to the resolute of the resultant of the two forces in that direction.

Proof

Let OA and OB represent the two forces $\underline{P}$ and $\underline{Q}$ and OC their resultant $\underline{R}$, so that OACB is a parallelogram. (See Fig. 5.16).

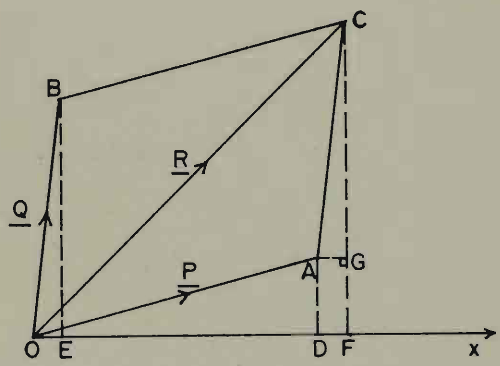


Fig. 2.16

Let OX be the given direction to which forces are to be resolved. Draw AD, BE, CF perpendicular to OX and AG perpendicular to CF.

The triangles OBE, ACG have their sides OB and AC equal, and their sides are parallel, so that they are equal in all respects.

$$OE = AG = DF$$

$$OF = OD + DF = OD + OE$$

R.H.S. of the above equation (i.e. OD + OE) represents the algebraic sum of the resolutes of the two forces $\underline{P}$ and $\underline{Q}$ in the direction OX, while L.H.S. represents the resolute of the resultant R in the direction OX.

Hence the theorem.

This theorem may obviously be extended to any number of forces. So the algebraic sum of the resolutes of the individual forces in a system of forces in a given direction is equal to the resolute of the resultant of the system of forces in that direction.

Following example will illustrate an application of the above theorem.

Example 2.8

ABC is an equilateral triangle. L,M and N are the mid points of AB, BC and CA respectively. A system of forces of magnitudes 4, $\sqrt{3}$, $2\sqrt{3}$, 3 and 5 Newtons acting at a point are represented in magnitude and direction by BA, BC, AM, LC and NB respectively. Find the resultant force and the angle it makes with BC.

Let us assume that the resultant R makes an angle θ with BC as shown in Fig. 1.18.

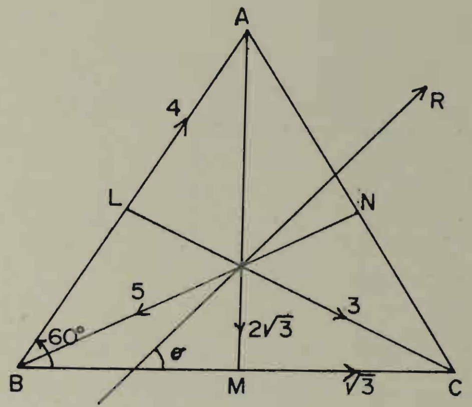


Fig. 1.18

Consider the resolution of forces in the BC direction. From the above theorem, we get,

$$R \cos \theta = \sqrt{3} + 4 \cos 60^{\circ} - 5 \cos 30^{\circ} + 3 \cos 30^{\circ}$$

Since the force $2\sqrt{3}$ is perpendicular to the BC, the resolute of it in the BC direction is zero.

$$R \cos \theta = \sqrt{3} + 4 \times \frac{1}{2} - 5 \times \frac{\sqrt{3}}{2} + 3 \times \frac{\sqrt{3}}{2}$$

$$= 2N \text{ --------(1)}$$

Similarly,

$$R \sin \theta = 4 \sin 60^{\circ} - 2\sqrt{3} - 5 \sin 30^{\circ} - 3 \sin 30^{\circ} \text{ N}$$

$$= 4 \times \frac{\sqrt{3}}{2} - 2\sqrt{3} - 5 \times \frac{1}{2} - 3 \times \frac{1}{2} \text{ N}$$

$$= -4\text{N} \text{ ---------- } (2)$$

Squaring both sides of the equations (1) and (2) and adding them together we get,

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = 2^2 + 4^2$$

$$R^2 (\cos^2 \theta + \sin^2 \theta) = 4 + 16$$

$$R^2 = 20 \quad \because \cos^2\theta + \sin^2\theta = 1$$

$$R = 4.47\text{N}$$

(2)/(1)

$$\frac{R \sin \theta}{R \cos \theta} = -\frac{4}{2}$$

$$\tan \theta = -2$$

$$\theta = 296.6^\circ$$

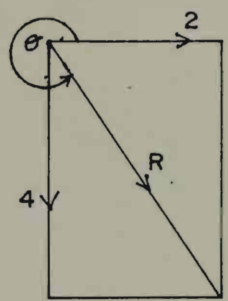


Fig. 2.19

The resultant is a force of 4.47 N, at angle 296.6° to BC.

S.A.Q. 2.2

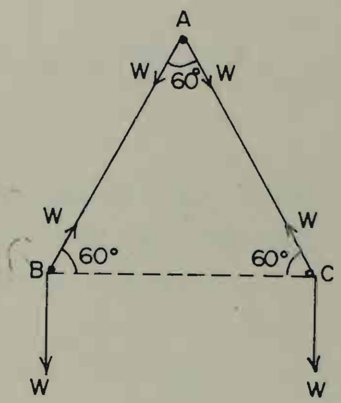
ABCD is a rectangle. AB and BC are of lengths 4 and 3 cm. A system of forces of magnitudes 3, 15, 2 and 10 Newton act along AB, BC, AD and DB respectively. Find the resultant force and the angle it makes with AB.

S.A.Q. 2.3

A system of forces of magnitudes x, $2\sqrt{3}$, 5 and 2 Newtons acts at a point O . The first force is horizontal. The rest of the forces are inclined to the first force at angles of 60°, 90° and 150° respectively. If the resultant force is of magnitude $3\sqrt{10}$, find the value of x.

ANSWERS TO SELF ASSESSMENT QUESTIONS

S.A.Q. 2.1



Let A,B,C be the positions of the pegs. Since the pegs are smooth, the tension is the same throughout the string and equal to W.

The thrust on A is the resultant of the two tensions of W inclined at an angle of 60°. If R is the magnitude of this resultant,

$$R^2 = W^2 + W^2 + 2 \times W \times W \cos 60^{\circ}$$
$$= 3.W^2$$
$$R = W\sqrt{3}$$

The thrust at B is equal to the resultant of the two tensions of W inclined at an angle of 150°. If S is the magnitude of this resultant, then

$$S^2 = W^2 + W^2 + 2W \times W \cos 150^{\circ}$$
$$= W^2\left(2 - 2 \times \frac{\sqrt{3}}{2}\right)$$
$$S = W\sqrt{2 - \sqrt{3}}$$

$$\because \cos 150^{\circ} = \cos(180^{\circ} - 30^{\circ})$$
$$= -\cos 30^{\circ}$$
$$= -\frac{\sqrt{3}}{2}$$

Let the thrust S at B be inclined to AB by an angle θ.

$$\tan\theta = \frac{W \sin 150^{\circ}}{W + W\cos 150^{\circ}} = \frac{\sin 150^{\circ}}{1 + \cos 150^{\circ}}$$

Now since $\sin 150 = 2 \sin \frac{150}{2}^{\circ} \cos \frac{150}{2}^{\circ}$

$$= 2 \sin 75^{\circ} \cos 75^{\circ} \text{ and}$$

$$1 + \cos 150^{\circ} = 2 \cos^2 75^{\circ}$$

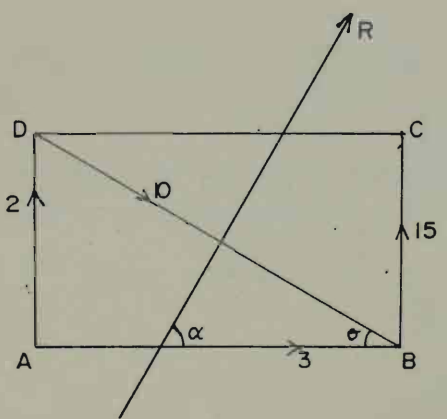
We have,

$$\tan\theta = \frac{2 \sin 75^{\circ} \cos 75^{\circ}}{2 \cos^2 75^{\circ}} = \tan 75^{\circ}$$

$$\theta = 75^{\circ}$$

We observe that the resultant bisects the angle between the forces. This is a common result when the two forces are equal. We also observe that the system is symmetrical about the vertical line drawn through A. So the thrust at C must be the same as that at B, and would bisect the angle between the two tensions W at C.

S.A.Q. 2.2



Let the resultant R be at angle α to AB.

From the Pythagorus theorem

$$BD^2 = AB^2 + AD^2$$
$$= 4^2 + 3^2 = 16 + 9 = 25$$
$$BD = 5 \text{ cm.}$$

$\sin\theta = \frac{AD}{BD} = \frac{3}{5}$; and

$\cos\theta = \frac{AB}{BD} = \frac{4}{5}$

By resolving the forces in the horizontal (AB) direction, we get

$$R\cos\alpha = 3 + 10\cos\theta \quad N$$
$$= 3 + 10 \times \frac{4}{5} = 11N \text{ -----------(1)}$$

By resolving the forces in the AD direction, we get

$$R\sin\alpha = 2 + 15 - 10\sin\theta$$
$$= 2 + 15 - 10 \times \frac{3}{5}$$
$$= 11N \text{ ---------(2)}$$

(2)/(1);

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{11}{11}$$

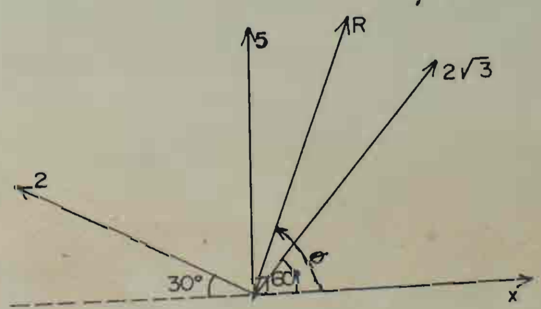
$$\tan\alpha = 1$$
$$\alpha = 45^{o}$$

$(1)^2 + (2)^2$ gives us,

$$R^2\sin^2\alpha + R^2\cos^2\alpha = 11^2 + 11^2$$
$$R^2(\sin^2\alpha + \cos^2\alpha) = 2 \times 11^2$$
$$R^2 = 2 \times 11^2$$
$$R = 11\sqrt{2}\ N$$

Hence the resultant is a force at magnitude $11\sqrt{2}$, making an angle 45^{o} with AB.

S.A.Q. 2 .3



Let the resultant R be inclined to the horizontal by an angle θ.

By resolution of forces, we get

$$R \cos \theta = x + 2\sqrt{3} \cos 60^\circ - 2 \cos 30^\circ$$

$$= x + 2\sqrt{3} \times \frac{1}{2} - 2 \times \frac{\sqrt{3}}{2}$$

$$= x \text{ --------(1)}$$

$$R \sin \theta = 5 + 2\sqrt{3} \sin 60^\circ + 2 \sin 30^\circ$$

$$= 5 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} + 2 \times \frac{1}{2}$$

$$= 9 \text{ ----------(2)}$$

Squaring and adding the equations (1) and (2), we get

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = x^2 + 9^2$$

$$R^2 (\cos^2 \theta + \sin^2 \theta) = x^2 + 81$$

Substituting $R = 3\sqrt{10}$ and using $\cos^2 \theta + \sin^2 \theta = 1$, we get

$$(3\sqrt{10})^2 = x^2 + 81$$

$$90 = x^2 + 81$$

$$x^2 = 9$$

$$x = \pm 3$$

Hence the magnitude of the force x is 3. Note that the plus or minus signs indicate the two possible directions of the force x along the positive x-axis or the negative x-axis.